

SELF-FOCUSING OF INTENSE LIGHT BEAMS

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A number of theoretical and experimental papers have been recently devoted to self-focusing of wave beams in nonlinear media. The phenomenon itself was first observed in [1]. Later experiments (see, e.g., [2-6]) were aimed primarily at establishing the existence of self-focusing (i.e., narrowing of the beam compared with the initial dimensions, or the presence of filaments with a divergence that is reduced compared with the diffraction divergence). At the same time, attempts were made to describe theoretically the propagation of wave beams in nonlinear media [7-11]. The most interesting from the physical point of view, and at the same time the most difficult mathematically, was the problem of the propagation of a wave beam in a nonlinear medium with a specified initial (say, Gaussian) intensity distribution. The published results of its numerical solution (see, e.g., [9]) only confirm the physically clear conclusion that the beam is initially narrowed down, but does not determine the subsequent evolution of the phenomenon. At the same time, it was recently shown [12] that the hitherto-employed analytic methods were incorrect. Correct analytic results could be obtained [12] only near the boundary of the medium and only in a narrow range of values of the initial field. By the same token, the question of the complete picture of the phenomenon remained open, and it was necessary to resort to a rather accurate and complete numerical solution for its theoretical clarification.

We present here the results of a numerical solution of the problem of self-focusing of an axially symmetrical beam. The obtained picture of the phenomenon differs greatly from that considered earlier. The corresponding solution has been obtained in a sufficiently large region encompassing the entire self-focusing process. The calculations were made for a Gaussian initial intensity distribution; the initial phase front was assumed plane, $E(r, 0) = E_0 \exp[-(r^2/2a^2)]$. The starting point was the well known parabolic equation (see, e.g., [11])

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} + 2ik \frac{\partial E}{\partial z} + n_2 k^2 |E|^2 E = 0, \quad (1)$$

first reduced to the dimensionless form

$$\frac{\partial^2 X}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial X}{\partial r_1} + 2iN \frac{\partial X}{\partial z_1} + N^2 |X|^2 X = 0, \quad (2)$$

where $r_1 = r/a$, $z_1 = z/\ell_x$, $\ell_x = a(n_2 E_0^2)^{-1/2}$, $X = E/E_0$, $N = E_0/E_{cr}$, and $E_{cr} = [n_2(ka)^2]^{-1/2}$ is the critical field in the initial section.

The equation (2) with boundary conditions $(\partial X / \partial r_1)|_{r_1=0} = 0$ and $|X|$ bounded as $r_1 \rightarrow \infty$ and with initial condition $X(0, r_1) = \exp[-(1/2)r_1^2]$ was solved by the following method: Equation (2) was approximated by the implicit difference scheme

$$-2iN \frac{X_k^{n+1} - X_k^n}{r} = \frac{1}{kh^2} \left[\left(k - \frac{1}{2}\right) X_{k-1}^{n+1} - 2k X_k^{n+1} + \left(k + \frac{1}{2}\right) X_{k+1}^{n+1} \right] + N^2 |X_k^n|^2 X_k^{n+1}, \quad (3)$$

where $X_K^n = X(r_n, h_K)$. The condition that the solution be bounded as $r_1 \rightarrow \infty$ was replaced at some remote point $r_1 = h_K$ by the equivalent relation $\partial X / \partial r_1 = a(r_1)X$ (see [13]), yielding a difference relation of the form

$$X_K^{n+1} = a_K^{n+1} X_{K-1}^{n+1}. \quad (4)$$

The boundary condition at $r_1 = 0$ was analogously replaced by

$$X_1^{n+1} = a_1^{n+1} X_0^{n+1}. \quad (5)$$

Here a_K^{n+1} and a_1^{n+1} are numbers that are known for each fixed z_1 . The system of linear algebraic equations (3) - (5) was solved by a difference approximation method (see, e.g. [14]). We succeeded in proving the stability of the difference scheme and the convergence of the solution of the difference equation to the solution of the differential equation (if the latter has a sufficient number of derivatives).

The values of the parameter N (the ratio of the initial field to the critical one) were specified in the interval from 0 to 10. For each specified value of N we obtained a solution $X(r_1, z_1)$. The values of $|X|^2$ were taken on the axis (i.e., at $r_1 = 0$) and at several points $r_1 = 1$. It was established as a result that when $N \leq 1$ the square of the field intensity on the axis ($|X(0, z_1)|^2$) decreases monotonically when the beam propagates into the medium. When $N > 1$ the $|X(0, z_1)|^2$ reveals a maximum the position and height of which coincide (when $N - 1 \ll 1$) with the values determined in [12]. When $N \rightarrow N_1$ ($N_1 \approx 2$) this maximum moves an infinite distance away from the plane $z_1 = 0$. On going through the value N_1 , one intense maximum remains on the $|X(0, z_1)|^2$ curve, but now it returns from infinity with increasing N , approaching the boundary $z_1 = 0$. On going through some succeeding value $N_2 > N_1$, a second

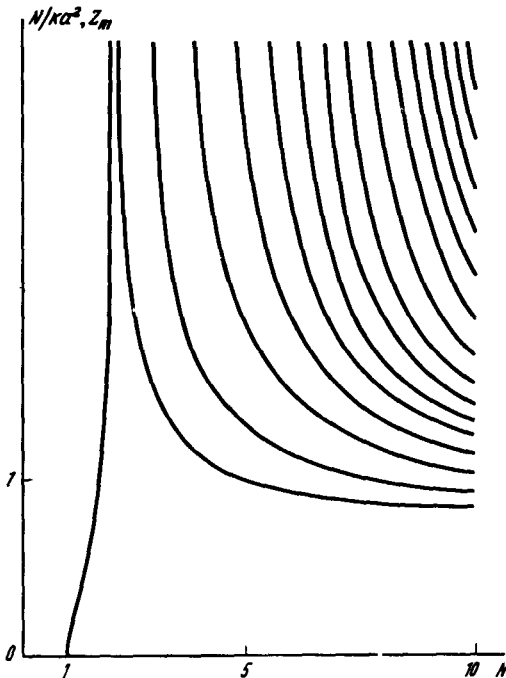


Fig. 1

intense maximum of the axial field appears and moves, like the preceding one, from infinity to the boundary $z_1 = 0$ with increasing N . There are similar values $N_3 < N_4 < N \dots$, passage through which gives rise to a maximum on the $|X(0, z_1)|^2$ curve; this maximum moves from infinity to the boundary $z_1 = 0$.

Thus, for any fixed $N > N_1$ the axial field has a finite set of intense maxima. The calculations determined their positions reliably along the beam axis. At the same time, the values of the maxima were not yet obtained with any accuracy (except that they are quite large). The number of these maxima and their arrangement on the beam axis z depend on N . The corresponding dependence and the general character of the arrangement are illustrated by the curves of Fig. 1. An essential fact is that only a fraction of the initial beam power is contained in each maximum (the fraction decreases with increasing number of maxima). An analysis of the field away from

the axis confirms the assumption that the beam acquires during the course of its propagation an "annular" structure such that each maximum is obtained by focusing an appropriate annular region. The latter statement is illustrated by the scheme of Fig. 2. It explains the fact that the intervals between neighboring maxima are much smaller than the distance from the boundary $z_1 = 0$ to the first maximum.

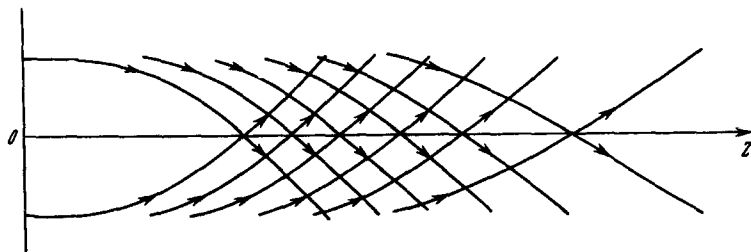


Fig. 2

An important role is played in the entire self-focusing process by the complex variation of the beam shape, and this determines in final analysis the very complicated character of the phenomenon. There is apparently a possibility of controlling the self-focusing process by selecting the initial distribution of the beam.

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MODE INTERACTION IN AN INJECTION SEMICONDUCTOR LASER

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Excitation of internal modes in semiconductor lasers can raise the threshold, lower the efficiency and the power, and lead to other interactions between the different modes. A convenient object for the investigation of the influence of internal modes on the operation of