the axis confirms the assumption that the beam acquires during the course of its propagation an "annular" structure such that each maximum is obtained by focusing an appropriate annular region. The latter statement is illustrated by the scheme of Fig. 2. It explains the fact that the intervals between neighboring maxima are much smaller than the distance from the boundary $z_1 = 0$ to the first maximum.

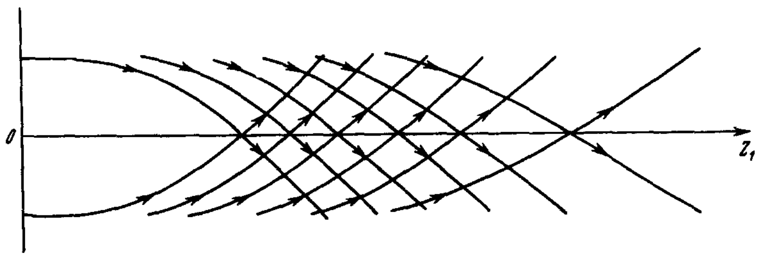

Fig. 2

An important role is played in the entire self-focusing process by the complex variation of the beam shape, and this determines in final analysis the very complicated character of the phenomenon. There is apparently a possibility of controlling the self-focusing process by selecting the initial distribution of the beam.

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MODE INTERACTION IN AN INJECTION SEMICONDUCTOR LASER

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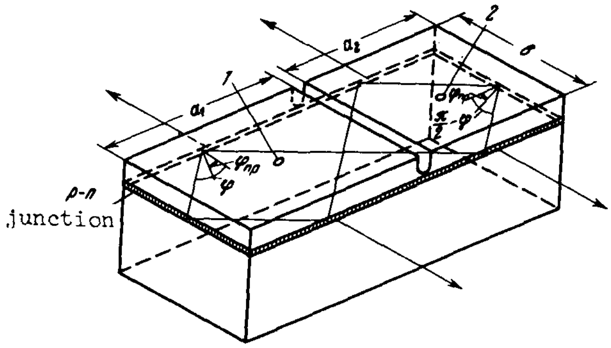
Excitation of internal modes in semiconductor lasers can raise the threshold, lower the efficiency and the power, and lead to other interactions between the different modes. A convenient object for the investigation of the influence of internal modes on the operation of of an injection laser was a GaAs laser diode with reflecting cleaved surfaces on all four sides [1] and with two insulated injection regions 1 and 2 (Fig. 1). To simplify the analysis,

Fig. 1. Semiconductor laser with four reflecting surfaces and two isolated injection regions 1 and 2 (the resonator length is $a = a_1 + a_2$ and its width is b).



it is convenient to separate in the spectrum of the natural oscillations of such a resonator three basic groups of modes: 1 - longitudinal modes with wave vector directed along the major axis of the crystal, 2 - transverse modes with wave vector perpendicular to the major axis, and 3 - internal modes with wave-vector directions satisfying the condition of total internal reflection.

The Q factors of the longitudinal, transverse, and internal modes are all different and proportional to the corresponding time constants:

$$\tau_a = [\tau_0^{-1} - \frac{c}{a}\ln R]^{-1}, \quad \tau_b = [\tau_0^{-1} - \frac{c}{b}\ln R]^{-1}, \quad \tau_0 = [c\alpha]^{-1} \qquad (1)$$

(α is the absorption coefficient and c the speed of light in the resonator material, R is the reflection coefficient, and a and b are the length and width of the resonator; if $a > b$, then $\tau_b < \tau_a < \tau_0$).

The expected picture of the interaction between the different modes in such a generator is as follows: If the injection is effected in only one of the regions (say 1), transverse oscillations are excited in it, but not longitudinal or internal, since they encompass also the non-injected region 2 and are strongly absorbed. If now a relatively small injection current is allowed to flow into region 2, then oscillations of the high-Q internal modes are produced and compete with the transverse oscillations, which they suppress. Thus the drop in the emission intensity of the transverse modes is evidence of the presence of internal oscillations, which cannot be observed directly. Further increase of the injection into region 2 causes complete suppression of the transverse oscillations in region 1 and further excitation of the transverse modes of region 2.

For an analysis of these phenomena we can assume a band model that has given good account of itself in comparisons with the experimental data, viz., a constant density of states ρ_B = const in the valence band, and an exponential energy dependence $\rho = \rho_0\exp(E/E_0)$ in the conduction band. Then, assuming in accord with [2] that the momentum selection rules do not hold in the exponential "tails," we can write for the negative-absorption coefficient at the temperature T = 0

$$g(\hbar\omega) = (cA)^{-1}\exp\frac{\hbar\omega}{E_0}[1 - \exp(-\frac{\mu_B}{E_0})] \quad (\hbar\omega < \mu), \tag{2a}$$

$$g(\hbar\omega) = (cA)^{-1}\exp\frac{\hbar\omega}{E_0}[\exp\frac{\mu - \hbar\omega}{E_0} - \exp(-\frac{\mu_B}{E_0})] \quad (\hbar\omega > \mu), \tag{2b}$$

where $\hbar\omega$ is the photon energy, μ and μ_B are respectively the Fermi quasilevels of the electrons and holes in the conduction and valence bands, reckoned from the top of the latter, and

$$A = \frac{m^2 (c_0/c)^2 \omega}{4\pi^2 e^2 E_0 \rho_0 \rho_B \langle|M|^2\rangle} \simeq \text{const} \tag{3}$$

(m and e are the mass and charge of the electron, c_0/c the refractive index, and $\langle|M|^2\rangle$ the mean square of the transition matrix element).

Under the same assumption, the spontaneous recombination rate in a unit volume is

$$S = A_0 \exp(\mu/E_0), \tag{4}$$

where

$$A_0 \simeq \frac{\omega^2}{A}\frac{\mu + \mu_B - E_0}{\pi^2 c^3 \hbar} \simeq \text{const}. \tag{5}$$

The stationary solutions of the system of kinetic equations, with allowance for injection and spontaneous and induced recombination, together with the stationary-threshold conditions of the type $gc\tau = 1$, lead to the following characteristic relations, which involve the dimensionless injection-current densities j, equal to the number of electron-hole pairs produced in a unit time in a unit volume of the active layer, and in which it is also assumed that the volume photon density N of a given mode is uniform over the entire resonator and at the same time regions 1 and 2 are completely insulated from each other with respect to the injection current.

1. In the case of injection in only one of the regions, say the first ($j_1 > 0$, $j_2 = 0$), there are excited transverse oscillations of frequency $\omega_b = \mu_1/h$ (μ_1 is the Fermi electronic quasilevel in region 1) and with photon density

$$N_b = \tau_b[j_1 - j_{1b}], \tag{6}$$

where

$$j_{1b} = \frac{AA_0}{\tau_b}[1 - \exp(-\frac{\mu_B}{E_0})]^{-1} \tag{7}$$

is the threshold current density.

2. If $j_1 \geq j_{1b}$, then an increase of the injection into the second region leads, if $j_2 \geq j_{20}$, to excitation of internal modes with simultaneous drop of the transverse-oscillation intensity

$$N_b = r_b[j_1 - j_{1b} - \beta(j_2 - j_{20})], \tag{8}$$

where j_{20} is the threshold injection-current density in the second region for the internal modes. Its value and that of the parameter β (ratio of negative-absorption coefficients in the first and second regions at the frequency ω_0 of the internal oscillations) are

$$j_{20} = j_{1b}\{1 - (1-\beta)[1 - \exp(-\frac{\mu_B}{E_0})]\}, \ \beta = 1 - \frac{a}{a_2}(1 - \frac{r_b}{r_0}) < 1, \ \omega_0 = \mu_1/\hbar, \tag{9}$$

if $a_1/a > \exp(-\mu_B/E_0)$, and

$$j_{20} = j_{1b}(r_b/r_0), \quad \beta = 1, \quad \omega_0 = \mu_2/h, \tag{10}$$

if $a_1/a < \exp(-\mu_B/E_0)$.

In either case $\beta > 0$ and an increase in j_2 gradually leads, at $j_2 = \overline{j_{2b}}$, to total extinction of the transverse oscillations by competing internal-mode oscillations (soft competing-mode regime), where

$$\overline{j}_{2b} = \beta(j_1 - j_{1b}) + j_{20}. \tag{11}$$

2. On the other hand, if $\beta < 0$, as is the case when $a_1/a > \exp(-\mu_B/E_0)$ and simultaneously

$$1 - (1 - \frac{r_b}{r_0})[1 - \exp(-\frac{\mu_B}{E_0})] > \frac{a_1}{a} > \frac{r_b}{r_0},$$

then there are no stationary solutions above the threshold $j_2 > j_{20}$ and the internal modes increase rapidly while the transverse oscillations disappear (hard competing-mode regime); this process ends when the photon density of the internal modes reaches the level

$$N_0 \simeq r_0[\frac{a_1}{a} j_1 - \frac{r_b}{r_0} j_{1b}]. \tag{12}$$

4. It should be noted that for any ratio of the parameters, an increase of j_2 after complete extinction of the transverse oscillations in region 1 brings about an increase of the level μ_2 until the threshold conditions are satisfied for the transverse oscillations in region 2, exciting the latter. All the formulas derived above remain in force, provided an appropriate permutation of the indices is made.

At other ratios of the parameters, which can be readily obtained from the same basic formulas, longitudinal oscillations can also be excited.

For convenience in comparison with experiment, Fig. 2a shows plots of formulas (9) - (11). Figure 2b shows, in the same coordinates, experimental results on two GaAs laser diodes with diffusion p-n junctions. The experiments were carried out under pulsed conditions at nitrogen temperature, using photomultipliers for the registration.

We see that the experimental points lie satisfactorily on straight lines similar to the theoretical ones of Fig. 2a (in spite of the difference in temperature between the calculation and the experiment). Judging from the slope of the total-suppression line $\overline{j}_{1b}$, the case

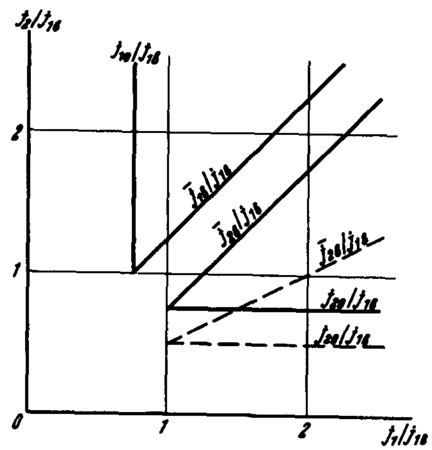


Fig. 2a. Plots of Eqs. (9) and (11) (dashed lines) and of Eqs. (10) and (11) (solid lines).

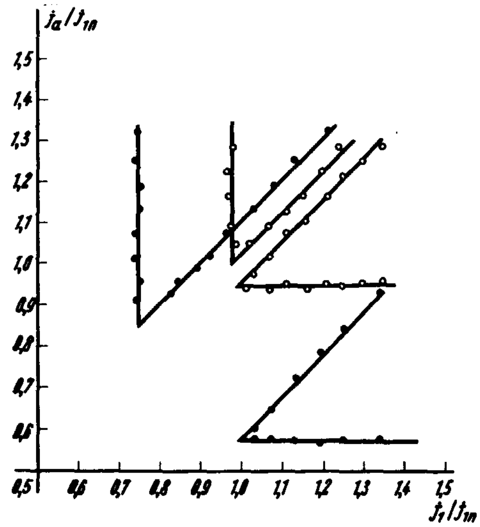


Fig. 2b. Experimental plots in the same coordinates as in Fig. 2a (• - sample with resistance between regions 1 and 2, $R_{1,2}$ = 1 ohm, $j_{1b} = j_{1n}$ = 4500 A/cm^2; o - $R_{1,2}$ = 20 ohm, $j_{1b} = j_{1n}$ = 3600 A/cm^2; $a_1/a = a_2/a = 0.5$.

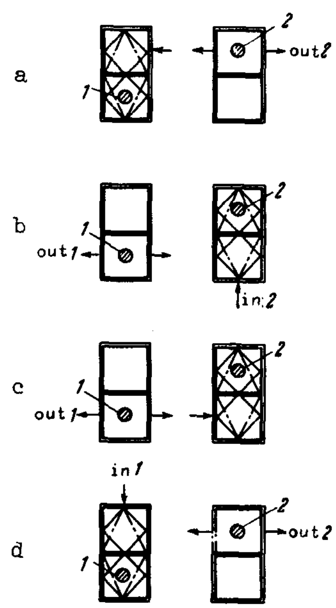


Fig.3. Diagram of optical trigger made up of a pair of diodes with four-surface resonators and insulated injection regions 1 and 2. a,c - stable states: emission of one diode maintains internal modes in the other; b, d - transients produced by light signal et 1 or 2.

realized is $a_1/a < \exp(-\mu_B/E_0)$, i.e., $\mu_B/E_0 < 0.7$, which is comparable with the data of [2]. It follows from the ratios j_{20}/j_{1b} and j_{10}/j_{2b} that τ_b/τ_a fluctuates for different resonators between 0.6 and 0.95, and accordingly the value obtained for α, on the order of several dozen reciprocal centimeters, does not contradict the experimental data.

It is important to note that the described procedure has made it possible to observe excitation of internal modes in diodes with one pair of cleaved surfaces and another pair of side surfaces produced by the usual technology (abrasive sawing).

When such diodes are additionally roughened, the threshold is lowered and the power is raised, sometimes by 1.5-2 times.*

The mode competition phenomenon can be used to control laser emission. From this point of view, special interest attaches to the hard competition regime (β<0), in which a weak control signal (according to (9), its level can be made quite low) produces rapid trigger switching of radiation with intensity N_b [Eq. (6)]. The bleaching of region 2, required to effect this switching, can be realized not only by the injection current j_2, but also by an optical signal from another laser. In this case a pair of optically coupled diodes with four surfaces form a high-speed trigger with two stable states (Fig.3), controlled by light signal and using the current injection only as a constant source of supply.

The authors are grateful to V. I. Magalyas for developing the experimental diodes and to Yu. V. Romanov for taking part in the measurements.

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*An increase in power by roughing was noted in [3].

INFLUENCE OF EXCHANGE NARROWING OF EPR ON THE DYNAMIC POLARIZATION OF NUCLEI

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It is shown in a number of papers [1], by means of a quantum-statistical analysis, that dipole-dipole (d-d) interaction between the electron spins exerts a strong influence on the dynamic polarization of nuclei (DPN). In the simple theory the DPN is obtained by saturation of forbidden resonances (FR) at frequencies $\omega^{\pm} = \omega_s \pm \omega_z$, so that the frequency spacing between the polarization maxima (of opposite signs) is equal to $D = 2\omega_I$ (ω_s and ω_I are the frequencies of the electron and nuclear resonances). In the rigorous theory of DPN [1] D can have values smaller than $2\omega_I$. It has been recently established by experiment [2] that D is much smaller than $2\omega_I$ in certain substances containing free radicals, and other anomalies have also been observed, which cannot be understood if only d-d interaction between electrons is taken into account. We show in this note that simultaneous allowance for exchange interaction (H_{ex}) and d-d interaction between the electrons improves greatly the agreement between theory and experiment.

Let us consider the case of strong fields, when the Zeeman electron energy H_z is much higher than their exchange and d-d energies, and the influence of the nonsecular part of the d-d interaction can be neglected. Under these conditions one should choose as a separate subsystem a system with Hamiltonian $H_e = H_{ex} + H_d$, which we shall call the exchange reservoir (ER). H_d is the secular part of the d-d interaction. The problem under consideration is equivalent mathematically to the problems solved in [1]. In the case of fast spin diffusion, the nuclear Zeeman subsystem (NZS) is characterized by a single reciprocal temperature β_I, which is defined in the stationary state by the formula

$$\beta_I = \frac{\beta_L}{2W_0(\Delta)T_s(\alpha\omega_e^2 + \Delta^2) + \alpha\omega_e^2}\left\{2\omega_s W_0(\Delta)T_s\Delta + \frac{\omega_s}{\omega_I}\frac{\alpha\omega_e^2[W(\omega^-) - W(\omega^+)]}{1/T_I + W(\omega^-) + W(\omega^+)}\right\}. \quad (1)$$

Here ω_e is the frequency corresponding to the interaction H_e, $\alpha = T_s/T_e$, where T_s and T_e are the spin-lattice relaxation times of the electronic Zeeman subsystem and the ER. T_I is the time