in the region  $r = r_0$ , for different values of the spacing between the terms  $2a^{-1}$ , determined from  $\epsilon_0$  and  $b_0$ ; F contains the centrifugal force, which can be readily shown to amount to 20 - 30% of  $F_{1,2}$  when  $E = E_0$ .

Unfortunately, there are no formulas for the calculation of the cross sections in a wide range of variation of  $\epsilon$  and b, making it impossible to compare the experimental curve with the calculation for  $E < E_0$ .

We are sincerely grateful to Professor V. M. Dukel'skii for interest in the work and A. N. Kozyrev for help with the calculations with the BESM-4 computer.

- [1] V. B. Matveev, S. V. Bobashev, and V. M. Dukel'skii, Zh. Eksp. Teor. Fiz. 55, 781 (1968) [Sov. Phys.-JETP 28 (1969)].
- [2] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Fizmatgiz, 1963 [Quantum Mechanics, Addison-Wesley, 1958].
- [3] V. K. Bykhovskii, E. E. Nikitin, and M. Ya. Ovchinnikova, Zh. Eksp. Teor. Fiz. 47, 750 (1964) [Sov. Phys.-JETP 20, 500 (1965)].
- [4] F. T. Smith, R. P. Marchi, W. Aberth, D. C. Lorents, and O. Heinz, Phys. Rev. <u>161</u>, 31 (1967).

## SPIN WAVES NEAR THE CURIE POINT

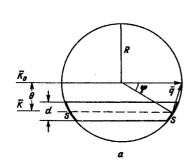
G. M. Drabkin, A. I. Okorokov, E. I. Zabidarov, and Ya. A. Kasman A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences Submitted 10 February 1969
ZhETF Pis. Red. 2, No. 6, 347 - 352 (20 March 1969)

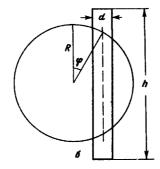
In the study of the critical scattering of neutrons in nickel [1], we observed flat maxima at  $T < T_c$ . With increasing angle of observation, these maxima shift towards higher

temperatures (these maxima were called in [1] scattering of type II). In scattering in iron, similar maxima were observed by Jacrot [2].

In the present paper we show that these maxima are due to scattering of neutrons by spin waves.

As is well known [3], in the case of a quadratic dispersion of the spin waves, the scattering of neutrons occurs in a cone with apex angle  $2\theta_{0}$ . The angle  $\theta_{0}$  does not depend on the





energy of the incident neutrons and equals  $1/\alpha$ , where  $\alpha = 2mA/\hbar^2$  (m - neutron mass, A - constant in the spin wave dispersion law,  $E_q = Aq^2 = A(\vec{k}_0 - \vec{k})^2$ ). The differential cross section of single-magnon scattering is given in this case by

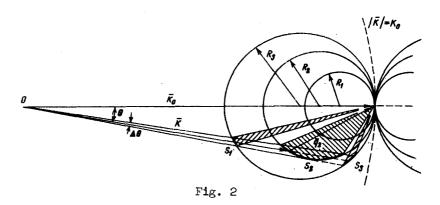
$$d\sigma/d\theta \sim (\theta_0^2 - \theta^2)^{-1/2}$$
 (1)

We recall also that a quadratic disper-

Fig. 1

<sup>1)</sup> For comparison, we determined the forces acting in the  $\text{K}^+$  + He collision at the point  $r=r_0$ , where  $U(r_0)=127.5$  eV. We use for the estimate the potential for the Ar + He pair [4], which yields  $F_0=9$  atomic units  $(r_0=0.69 \text{ A})$ .

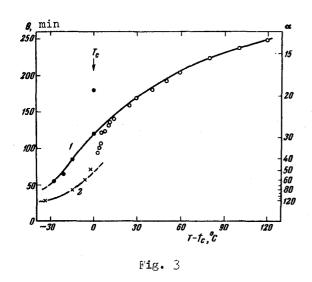
sion law can be used if the condition  $E > 2\mu_B \alpha H$ ,  $4\pi \mu_B \alpha M_O$  is satisfied [3], where E - energy of incident neutrons, H - external magnetic field, and  $M_O$  - saturation magnetization. These conditions are satisfied in the cases considered below.



When T is approached

in the region T < T\_{c}, the quantity A, and consequently also the effective exchange integral, decreases [4,5]. This means that the scattering surface increases with increasing temperature; this surface constitutes a sphere of radius  $l\alpha(T) = \theta_{0}(T)$ . If one fixes in the experiment the angle  $\theta$  such that  $\theta > \theta_{0}(T_{1})$ , where  $T_{1} \ll T_{c}$ , then no scattering takes place when  $T = T_{1}$ . If the temperature is increased, then at a certain  $T = T_{m}$  the quantity  $\theta_{0}(T_{m})$  becomes equal to the observation angle  $\theta$ ,  $(\theta_{0}(T_{m}) = \theta)$ . In this case, obviously, the intensity of the spin-wave scattering will be maximal, in accordance with formula (1). If we take into account the divergence  $\Delta\theta$  of the incident-neutron beam and the angular width of the detector (for brevity, we can include  $\Delta\theta$  in the effective detector with d), then the scattering intensity will change from zero to its maximum value when  $\theta_{0}$  changes from  $\theta$  - (d/2) to  $\theta$  + (d/2). With further increase of  $\theta_{0}$  the intensity will decrease, roughly speaking, in accordance with formula (1). Thus, from the position of the maximum of neutron scattering through a specified angle we can determine  $\theta_{0}(T_{m})$  and consequently  $\alpha(T_{m}) = 1/\theta_{0}(T_{m})$ , and by the same token  $A(T_{m})$ .

Let us explain in somewhat greater detail the conditions for setting up an experiment for observing this maximum. Figures 1 and 2 explain the scattering geometry. The observed intensity is proportional to the area S of intersection of the beam with the scattering surface of radius  $R = \theta_0$  (Fig. la). When  $d \ll R$  and  $h \ll R$  we have  $S \sim dh[1 - (\theta^2/\theta_0^2)]^{-1/2}$ for  $d/2\theta_0 < \theta/\theta_0 < [\theta_0 - (d/2)/\theta_0]$ , and the intensity increases with increasing  $\theta/\theta_0$ . As follows from Fig. 1b, when d  $\ll$  R and h >> 2R the effective height reached by the scattered neutrons decreases with increasing  $\theta/\theta_0$  like  $h_{\rm eff} \sim [1 - (\theta/\theta_0^2)]^{1/2}$ , and the counting rate will be constant within the limits of the scattering cone. A measurement procedure with h  $\gg$  2R was used in [6] to determine  $\theta_0$ . In [1], the angular height of the detector was h=20 min, and the angles at which the scattering maxima were observed were  $\theta_m=24$  - 67 min. Such experimental conditions correspond to the intermediate case with h < 2R. Figure 2 shows that the scattering maximum corresponds to the area  $S_{\rho}$  of the intersection of the beam with the scattering sphere  $R_2$ , since  $S_2 > S_1 + S_3$ , where  $S_1$  and  $S_3$  is the analogous area for the sphere R3. The surface R1 takes no part at all in the scattering. When the temperature is varied and  $\theta$  is fixed, the radius of the sphere R(T) can go through all the values of R, namely,  $R_1$ ,  $R_2$ , and  $R_3$ . The observed form of the peak obviously depends on the relations



between  $\theta$ ,  $\theta_0$ , h, and d, and in general will not be symmetrical on the two sides of the maximum: when  $T < T_m$  the slope of the descent is proportional to  $(d/\theta)^{-1}(d\theta_0/dT)$ , and when  $T > T_m$  it is proportional to  $-[(\theta_0^2/\theta^2) - 1]^{-3/2}(d/\theta)^{-1} \times (d\theta_0/dT)$ .

Figure 3 shows the experimentally obtained temperature dependence  $\theta_{\rm m}(T)$ , plotted for  $T < T_{\rm c}$  from the data of Jacrot [2] for iron and our data [1] for nickel. (The points for nickel have been corrected for the angular resolution). We see that  $\alpha$  decreases when T approaches  $T_{\rm c}$ . We note in this connection that the decrease of  $\alpha$  with increasing temperature was observed earlier by Riste [7] in magnetite. The experimentally

The experimentally observed width of the peaks decreases when  $T \to T_c$  [1, 2], which agrees with the estimates given above, since the rate of change of  $\alpha(T)$  increases when  $T \to T_c$  (Fig. 3).

It is obvious that when T is close to  $T_c$  the scattering of type II is superimposed on the critical scattering (scattering of type I in accordance with the terminology of [1], which has at  $T = T_c$  a maximum independent of the observation angle. Therefore, in the vicinity of  $T_c$ , experiments of this type do not make it possible to determine  $\alpha(T)$  for spin waves 1. In Fig. 3 this region is characterized by a spread of the points away from the smoothly drawn curve, since these points correspond to the maxima of the total scattering.

It follows from Riste's results [7] that the single-magnon scattering by magnetite, which he investigated, does not disappear even when T >  $T_c$ . We have attempted to construct a similar  $\alpha(T)$  plot from the results of Bally [10], where a shift of the maximum of the critical scattering by iron was observed at T >  $T_c$ . Apart from a small region near  $T_c$ , the experimental points fitted a smooth continuation of curve 1 drawn in accordance with the data of Jacrot [2]. We can therefore propose that the maxima observed [10] at T >  $T_c$  are also due to single-magnon scattering. The fact that in [2, 10] they used different neutron energies ( $\lambda$  = 4.75 Å at T <  $T_c$  [2] and  $\lambda$  = 1.25 Å at T >  $T_c$  [10]) favors the assumption that the scattering has a spin-wave nature, since the radius of the scattering sphere  $\theta_0$  does not depend on the energy of the incident neutrons. It is interesting to note that the shape of the peak observed by Bally at angles close to 3° agrees with the foregoing estimates: when T <  $T_m$  the slope of the peak is steep, and when T >  $T_m$  it is gentle. The abrupt vanishing of scattering at T  $\approx$  910° C [10] is apparently connected with a jumplike change of  $\alpha(T)$  during the structural transition in iron (from a bcc to an fcc lattice).

<sup>1)</sup> 

Here and below we speak of spin waves at  $T \geq T_c$ , although the question of their existence at this temperature remains open; it is thoroughly discussed in a number of papers [8, 9].

If we assume that the type II scattering at T < T  $_{\rm c}$  and T > T  $_{\rm c}$  is scattering by spin waves excited within the correlation region with nonzero magnetization, then this scattering is expected to exist also at  $T \simeq T_0$ . From curve 1 and 2 (Fig. 3) we can conclude that the maximum of such a scattering will coincide with  $T_c$  at angles  $\theta$  =  $\theta_s$   $\approx$  2° for iron and  $\theta_s$   $\sim$  70 min for nickel. This means that near these observation angles it is necessary to approach the results of an analysis of the critical scattering of neutrons with caution.

The inelasticity of the critical scattering at  $T = T_c$ , which was observed in many investigations [2, 11, 12], can apparently be due, to a considerable degree, to an admixture of spin-wave scattering. To the contrary, our experiments [13] and the data of [14] point to a quasielastic character of the scattering in the immediate vicinity of T. In [13] this result was obtained by analyzing the polarization of the neutron scattered through an angle  $\theta$  = 10.2 min, which is far from  $\theta_{g}$  for nickel. In [14], such a result was obtained from an energy analysis of the scattered neutrons in terbium, where the spin-wave excitations are apparently insufficiently well developed, owing to the strong anisotropy of this ferromagnet.

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- [1]
- G. M. Drabkin, E. I. Zabirov, Ya. A. Kasman, and A. I. Okorokov, Zh. Eksp. Teor. Fiz. 56, 479 (1969) [Sov. Phys.-JETP 29 (1969)].
  B. Jacrot, J. Konstantinovich, G. Paratte, D. Cribier, Inelastic Scattering of Neutrons in Solids and Liquids) IAEA, Vienna, 1963), v. II, p. 317.
  S. V. Maleev, Zh. Eksp. Teor. Fiz. 48, 1448 (1965) [Sov. Phys.-JETP 21, 969 (1965)].
  V. G. Vaks, A. I. Larkin, and S. A. Pikin, ibid. 53, 1089 (1967) [26, 647 (1968)].
  M. Hatherly, K. Hirakava, R. Lowde, and F. Mallett, Phys. Rev. 86, 318 (1952).
  M. Hatherly, K. Hirakava, R. Lowde, F. Mallett, M. Stringfellow, and B. Torrie, Proc. Phys. Soc. 84, 55 (1964) [2]
- [3]
- [4]
- [5]
- [6] Phys. Soc. 84, 55 (1964).
- [7] T. Riste, J. Phys. Soc. Japan 17, Suppl. B-III, 60 (1962).
- [8] T. Riste, J. Appl. Phys. 39, 528 (1968).
- J. L. Beeby and J. Hubbard, Phys. Lett. 26A, 376 (1968). [9]
- [10]
- [11]
- D. Bally, B. Grabcev, M. Popovici, M. Totia, and A. M. Lungu, J. Appl. Phys. 39,459(1968)
  D. Cribier, B. Jacrot, and G. Parette, J. Phys. Soc. Japan 17, Suppl. B-III, 67 (1962).
  L. Passell, K. Blinowski, T. Brun, and P. Nilsen, J. Appl. Phys. 35, 933 (1964);
  Phys. Rev. 139, 1870 (1965). [12]
- G. M. Drabkin, E. I. Zabidarov, Ya. A. Kasman, and A. I. Okorokov, ZhETF Pis. Red. 2, 541 (1965) [JETP Lett. 2, 336 (1965)]. [13]
- [14] J. Als-Nielsen, O. W. Dietrich, W. Marshall, and P. A. Lingard, Solid State Comm. 5, 607 (1967).

## MOSSBAUER EFFECT IN INDIUM-GALLIUM IRON GARNET

G. N. Belozerskii, V. N. Gittsovich, and A. N. Murin Leningrad State University Submitted 12 February 1969 ZhETF Pis. Red. 9, No. 6, 352 - 356 (20 March 1969)

In a recent interesting paper [1], Gruzin and co-workers reported observation of four magnetic sublattices in indium-gallium iron garnet. We have investigated a garnet of similar composition, obtained from the same laboratory as in [1].

The NGR spectra were plotted with an electrodynamic setup in which the absorber was moved sinusoidally relative to the source. The detector was a proportional counter. The spectra were registered with a 400-channel analyzer and reduced by an electronic computer with