SUPERCONDUCTIVITY IN SYSTEMS WITH ARBITRARY INTERACTION SIGN

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1. The usually assumed necessary condition for the occurrence of superconductivity is that the sign of the effective interaction of the quasiparticles on the Fermi boundary (FB) be negative. Actually this condition is valid only for an interaction having a peak on the FB itself. Yet this peak may turn out to be shifted relative to the FB, by a distance exceeding its width. In this case, "pairing" of the quasiparticles on the FB is possible only as a result of second-order processes, which lead to an effective attraction regardless of the sign of the interaction, including the case of repulsion forces. In the present paper we clarify the quantitative aspect of this question and discuss several physical applications.

2. We start from the well-known equation for the gap [1]

$$\Delta(\xi) = -\frac{1}{2}\sum_{p'} V_{pp'}\frac{\Delta(\xi')}{\sqrt{\xi'^2+\Delta^2(\xi')}}, \qquad (1)$$

where $\xi = v_F(p - p_F)$; $V_{pp'}$ - effective interaction ($\vec{p}$ and $-\vec{p}$ are the initial and $\vec{p}'$ and $-\vec{p}'$ the final momenta of the quasiparticles). Referring all the slowly-varying functions to the FB, we get in the weak-coupling limit

$$\Delta(\xi) = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi'}{\sqrt{\xi'^2+\Delta^2(0)}}\Delta(\xi')U(\xi-\xi'),$$

where U is the interaction multiplied by the level density and averaged over the angles, and depends usually on the difference of the arguments only. In the language of the effective dielectric constant we have

$$U = a\int_{\omega/v_F}^{2p_F}(dk/k\epsilon(\omega,k)),\quad \omega = |\xi-\xi'|,\quad a = e^2/\pi v_F. \qquad (2)$$

The equation for the gap is obtained from (1):

$$\Delta(0) = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi}{\sqrt{\xi^2+\Delta^2(0)}}\Delta(\xi)U(\xi). \qquad (1')$$

The solution of (1) can be represented in the form

$$\Delta(\xi) = \Delta(0)\left[1-\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi'\chi(\xi,\xi')}{\sqrt{\xi'^2+\Delta^2(0)}}\right],$$

where χ is a solution, regular when $\Delta(0) \to 0$, of the equation

$$\chi(\xi,\xi') - U(\xi-\xi') + U(\xi') = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi''}{|\xi''|}\chi(\xi,\xi'')[U(\xi''-\xi') - U(\xi')] = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi''}{|\xi''|}[U(\xi-\xi'') - U(\xi'')]\chi(\xi'',\xi').$$

Substituting the expression for $\Delta(\xi)$ in (1') and introducing a new variable

$$\phi(\xi) = -U(\xi) + \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi'}{|\xi'|}U(\xi')\chi(\xi',\xi),$$

we obtain

$$1 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi}{\sqrt{\xi^2+\Delta^2(0)}}\phi(\xi). \qquad (1'')$$

Retaining in (1'') only the terms that are most singular in $\Delta(0)$, we arrive at a formula of the BCS type

$$\Delta(0)\sim\tilde{\omega}\exp(-1/g^2), \qquad (3)$$

$$g^2 = \phi(0),\ \ln\tilde{\omega} = -\frac{1}{g^2}\int_0^{\infty} d\xi \ln(2\xi)\,\phi'(\xi),$$

where ϕ is the solution of the equation

$$\phi(\xi) = -U(\xi) - \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi'}{|\xi'|}\phi(\xi')[U(\xi-\xi')-U(\xi)]. \qquad (4)$$

Superconductivity takes place when $g^2 > 0$, i.e., when[1)]

$$U(0) + \int_0^{\infty}\frac{d\xi}{\xi}\phi(\xi)[U(\xi)-U(0)] < 0. \qquad (5)$$

In the weak-coupling limit ($g^2 \ll 1$), Eq. (4) can be solved by iteration. If U has a peak of width $\Delta\xi$ at $\xi = 0$, then we can confine ourselves in (4) and (5) to the linear terms, and we obtain the usual BCS formula. In the opposite case, when the shift of the peak relative to the FB is large compared with $\Delta\xi$, we can put $U(0) = 0$. Then the second iteration of (4) is significant, and the condition (5) is satisfied automatically. As a result we have

$$g^2 = \int_0^{\infty}\frac{d\xi}{\xi}U^2(\xi), \qquad \tilde{\omega}\sim\Delta\xi. \qquad (6)$$

3. The second of the considered cases corresponds to the well-known "jelly" model [2, 3], for which $\varepsilon(\omega, k) = 1 - (\omega_0^2/\omega^2) + \kappa/k^2)$ (ω_0 is the plasma frequency of the ions and κ the Debye momentum. From (2) we get

$$U(\xi) = -\frac{1}{2x}\ln|1-x|, \quad x = \frac{1}{a}\left(\frac{\omega_0^2}{\xi^2}-1\right),$$

which vanishes on the FB and has tall but narrow peaks (width $\alpha\omega_0$, where $\alpha \ll 1$ in the weak-coupling limit) at a distance $\sim\omega_0$ from the FB. The formulas of the preceding section yield

$$\Delta(0)\sim a\omega_0\exp\left(-\frac{8}{\pi^2 a}\right); \qquad (7)$$

(the critical temperature T_c is connected with this quantity in the usual manner). The corresponding value of g^2 differs greatly from that obtained in the same model by de Gennes [3] ($g^2 \approx 3Z^{2/3}\alpha$, Z - atomic number), who started with excessively crude and unjustified approximations.

4. The de Gennes result was recently used by Ginzburg and the author [4] to estimate

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1) It is precisely in light of this condition that the results obtained in previously investigated models (see, for example, [1]) become understandable. In these models it was necessary to cope with the appearance of superconductivity under conditions when U (0) > 0.

T_c in the substance of a cold white dwarf[1]. The correct formula (7) lowers this estimate by more than one order of magnitude (at a density on the order of 10 g/cm^3). It must be borne in mind, however, that the "jelly" model is applicable to a solid, if at all, only at extremely high compressions, when the contribution of the transverse modes and of the umklapp processes is suppressed. Moreover, even under these conditions the "jelly" model is incapable of describing the interaction at large momentum transfers, which play an important role in our case. Therefore the discussed estimate of T_c can vary in either direction. At any rate, it can be firmly asserted that superconductivity of the substance in the central part of a white dwarf of the ordinary type is completely excluded, owing to the exponential smallness of T_c (see (7) and (5))[2]. As to the superconducting regions on the periphery of cold white dwarfs, referred to in [4], their existence is quite possible.

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OPTICAL FREQUENCY STANDARD WITH NONLINEARLY ABSORBING GAS CELL

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1. In [1, 2] we proposed to obtain ultrahigh frequency stability of a laser by using a narrow Lamb dip in a resonantly-absorbing cell with a molecular gas at low pressure. In this case it is possible to stabilize the frequency against the power peak[3] with the aid of external feedback [2], or else obtain self-stabilization of the frequency as a result of the nonlinear frequency pulling [1]. Stabilization against the power peak was first realized in an He-Ne laser operating at λ = 6328 Å with an Ne absorbing cell [2, 4], and recently in an He-Ne laser at 3.39 μ with a CH_4 absorbing cell [5].

The purpose of the present paper is to indicate new possibilities of obtaining ultrahigh

[1] A general analysis of superconductivity of strongly compressed matter [5] does not lead to any unambiguous results.

[2] Just the opposite is concluded in a just-published paper [6], where a non-exponential formula, which we believe is wrong, was obtained for T_c. In the derivation of this formula the relative changes of the electron and ion densities are assumed equal, an assumption valid only in the case of strong coupling, $\alpha \geq 1$, if at all (in strongly compressed matter, to the contrary, $\alpha \ll 1$). As to the "jelly" model, which essentially is involved in the discussed paper, its correct application likewise leads to an exponential formula (see (7)).

[3] The power peak in an He-Ne laser with an absorbing Ne cell was observed in [3].