currents. Curves 2, 3, and 4, were calculated by Kopaleishvili et al. [5] they are normalized at a cross section value of approximately 0.17 mb, i.e., the absolute values of the cross sections up to maxima for these curves are reduced respectively by three, three, and four times. In this paper we calculated the total cross sections by using different nucleon-nucleon interaction potentials and the corresponding wave functions in curve 2. A velocity-dependent potential was used to calculate while curves 3 and 4 were calculated with a potential with a repulsion core. The correlation function was taken into account for curve 3.

As seen from the figure, our data point to certain new singularities in the behavior of the total cross section as a function of the γ-quantum energy:

1. The maximum of the curve is quite broad.

2. The curve reveals two peaks. The maximum of the first is located near 50 MeV and that of the second near 75 MeV. The cross sections at the maxima are 0.2 and 0.17 mb. From a comparison of our data with the results of other authors we see that the experimental curve given by Gorbunov et al. is in good agreement with the first peak. None of the theoretical curves describes the structure revealed by our experiments in the dependence of the toal reaction cross section of the γ-quantum energy. The theoretical curves 2, 3 and 4 differ from our data on the absolute magnitudes of the cross sections at the maxima.

The authors are grateful to A. N. Gorbunov for a discussion of the results and for valuable remarks.

[1] J. H. Smith and M. Q. Barton, Phys. Rev. 110, 1143 (1958).
[2] J. M. Reid and B. Lalovic, Proc. Phys. Soc. 76, 65 (1960).
[3] A. N. Gorbunov and V. M. Spiridonov, Zh. Eksp. Teor. Fiz. 34, 866 (1958) [Sov. Phys.-JETP 7, 600 (1958)].
[4] R. I. Dzhibuti, I. B. Krupennikova, and V. I. Mamasakhlisov, Yad. Fiz. 7, 803 (1968) [Sov. J. Nucl. Phys. 7, 489 (1968)].
[5] T. I. Kopaleishvili and I. Z. Machabeli, Izv. AN SSSR, ser. fiz. 30, 510 (1966).

SCATTERING OF X-RAYS BY HYDROGEN ATOMS

V. G. Gorshkov and V. S. Polikanov
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
Submitted 14 January 1969
ZhETF Pis. Red. 9, 464-468 (20 April 1969)

Recently Gavrila [1] obtained an expression for the elastic scattering of light by hydrogen in the dipole approximation. Zon, Manakov, and Rappoport [2] obtained independently more general formulas in the dipole approximation for the scattering with transitions between arbitrary shells of the atom without ionization. In the present paper we obtain formulas for the scattering of photons by hydrogen atoms at the frequency ω, including frequencies on the order of the average electric momentum in the atom $\eta = m\alpha Z$ ($\hbar = c = 1$).

We consider arbitrary transitions between the shells of the atom, including ionization. For simplicity we confine ourselves to the nonrelativistic region $\omega \ll m$.

The Compton effect on the bound electron is described by the diagrams in the figure, where the shaded blocks represent the interaction with the Coulomb field that does not change the energy but changes the momentum in the line. The energy-momentum conversation law is satisfied at all vertices. Integration is carried out over the intermediate momenta. Diagram c gives the classical Thompson limit as $\alpha \to 0$. In the vertices of the diagrams a and b we have the operator $\gamma = \vec{p}\vec{A} + \vec{A}\vec{p} + i\sigma([\vec{p}, \vec{A}] + [\vec{A}, \vec{p}])$, which has in momentum space the form

$$<f|\gamma|f'> = 2e(f + \frac{i}{2}[\sigma \times k])<f - k|f'>, \tag{1}$$

where $\vec{e}$ and $\vec{k}$ are the polarization and the momentum of the photon. The wave function of the bound state of the electron can be written in the form of the derivatives and gradients of the Yukawa potential:

$$<\psi_{n\ell m}|f - k> = \pi\sqrt{8}\Gamma^k_{n\ell m}(-\frac{\partial}{\partial\eta_n})<k|V_{\eta_n}|f>, \tag{2}$$

$$<k|V_\eta|f> = \frac{1}{2\pi^2}\frac{1}{(k - f)^2 + \eta^2}, \quad \eta_n = \frac{maZ}{n},$$

$$\Gamma_{n\ell m} = N_{n\ell m}(2\eta_n)^{3/2} L^{2\ell+1}_{n+\ell}(2\eta'_n\delta_n)(2\eta'_n\delta_n)^\ell x_1^m \frac{d^m P_\ell(x_0)}{dx_0^m}, \tag{3}$$

$$x_l \equiv \epsilon_l \vec{\nabla}_k/\delta_n, \quad \delta_n \equiv \partial/\partial\eta_n, \quad \Gamma_{100} = \eta_1^{3/2} \quad \Gamma_{200} = \eta_2^{3/2}(1 + \eta_2 \frac{\partial}{\partial\eta_2}),$$

$$\Gamma_{21m} = \eta_2'^{5/2}(\epsilon_m \vec{\nabla}_k),$$

where $N_{n\ell m}$ is the product of the normalization coefficients of the Laguerre polynomials and spherical functions. The unit vector $\vec{\varepsilon}$ (the direction of the spin of the vector atom with $\ell = 1$) takes on the values $\vec{\varepsilon}_0 = \vec{\varepsilon}_z$, $\vec{\varepsilon}_{\pm1} = \varepsilon_x \pm i\varepsilon_y)/\sqrt{2}$. The operator $-\vec{f}(\partial/\partial\eta)$, which results from the substitution of (1) and (2) in the diagrams a and b, is equivalent to the operator $\eta\nabla_k$. Taking the foregoing into account, we can write the amplitudes of any process without ionization in the form (the upper indices of the operators number the photons, and the lower ones the electrons; the spin functions have been omitted).

$$A = 8r_0 \left\{ 2(K_a^{21} + K_b^{12}) - (\vec{\ell}_1\vec{\ell}_2)\Gamma_2^2\Gamma_1^1 \frac{\eta_2 + \eta_1}{[(k_2 - k_1)^2 + (\eta_2 + \eta_1)^2)^2} \right. \tag{4}$$

where

$$K_a^{ii} = \gamma_2^i \gamma_2^i J_0^{ii}(p_a), \quad \gamma_\ell^i = \Gamma_\ell^i e_i(\eta_\ell \nabla_{k_i} + (-1)^\ell \frac{i}{2}[\vec{\sigma}k'_i]\frac{\partial}{\partial\eta_\ell}), \tag{5}$$

$$J_0^{ii}(p) = \pi^2 <k_i|V_{\eta_2} G_c^p V_{\eta_1}|k_i>, \quad \eta_\ell \equiv \eta_{n\ell}. \tag{6}$$

$$p_a = \sqrt{2m(\omega_1 - E_1) + i\epsilon}, \quad p_b = i\sqrt{2m(\omega_2 + E_1)}, \quad E_1 = \frac{\eta_1^2}{2m},$$

$$r_0 = \frac{a}{m}. \tag{7}$$

The derivatives in formulas (3) and (5) do not act on the primed variables. We see that any amplitude is determined by a matrix element with a Coulomb Green's function (6). This matrix element has been calculated in [3] and is given by the second term of formula (20b) in [3], divided by $2E\alpha Z$, if we put in it $\eta = \eta_2$, and Λ is replaced by expression (15a) with $a^2 = -\eta_1^2$ and $y = x$, or by formulas (25) and (28) of [3], from which the factor $2E$ and derivatives with respect to η_1 and η_2 should be omitted. Making the following change of variables in (20b) of [3]

$$\frac{dx}{x\Lambda} = \frac{dt}{t} \qquad \left(t = \frac{\zeta + 1}{\zeta - 1}\right),$$

we obtain

$$J_0^{21}(p) = \frac{1}{ip^3}\int_1^\infty \frac{t^{i\xi}\,dt}{at^2 - 2bt + a^*} = \frac{1}{ip^3 2\sqrt{b^2 - |a|^2}} \times$$

$$\times\ (\phi(x_+) - \phi(x_-)), \tag{8}$$

$$\phi(x) = -F(1, -i\xi, 1 - i\xi, x)/i\xi, \qquad x_\pm = (b \pm \sqrt{b^2 - |a|^2})/a, \tag{9}$$

$$a = a_1 a_2, \quad b = \beta_1\beta_2 - 4n_1 n_2, \quad \xi = m\alpha Z/p, \tag{10}$$

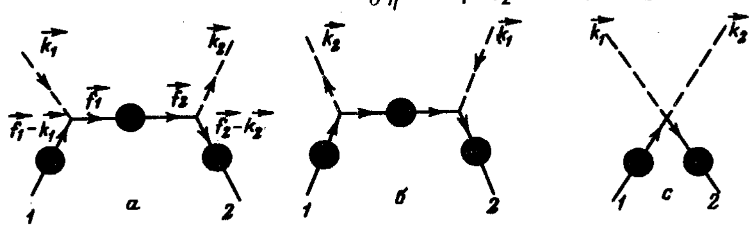
$$a_\ell = n_\ell^2 - (1 + i\mu_\ell)^2, \quad \beta_\ell = 1 + n_\ell^2 + \mu_\ell^2, \quad n_\ell = k_\ell/p, \quad \mu_\ell = \eta_\ell/p.$$

We note that diagram a has an imaginary part at $\omega_1 > E_1$, which equals the total cross section of the photoeffect from the corresponding shell in the case of forward elastic scattering. Diagrams b and c are real in the nonrelativistic approximation. The scattering cross section is connected with the amplitude (4) by the formula

$$d\sigma = \Sigma|A|^2 \frac{\omega_2}{\omega_1} d\Omega_{k_2}, \tag{11}$$

where the summation sign denotes averaging over the initial and summation over the final spin (magnetic) states. In the dipole approximation ($\omega \ll \eta$), our formulas go over into the formulas of Gavrila [1] and of Zon, Manakov, and Rappoport [2] for the transitions ns → ns and 1s → 2s. When $\omega \ll \eta$, we obtain from (4) the amplitude of the processes that are forbidden in the dipole approximation, for example the transition 1s → 2p. In a recent preprint, Fronsdal [4] obtained the amplitude of elastic scattering by the ground state of hydrogen at $\omega \sim \eta$ without allowance for the electron spin. Our formula differs from that of Fronsdal. The two formulas agree if we replace the last term $- \omega^2/\mu$ in the upper formula (34) of [4] by $- \omega^2/2\mu$. This term does not arise in the dipole approximation.

The amplitude for scattering with ionization of the atom can be obtained analogously by using the wave function of the emitted electron in the form (17a, c) of [5], formula (20b) or (28) of [3], and the identity (see (3)) $-\frac{\partial}{\partial\eta} V_{\eta_1} V_{\eta_2} = V_{\eta_1 + \eta_2}$.



$$A_1 = 8r_0N_2\{\ 2(M_a^{21} + M_b^{12}) + \frac{1}{2}\Gamma_1^1 \frac{\partial}{\partial\eta_1} \frac{(e_1 e_2)}{(q + p_2)^2 + \eta_1^2} \times$$

$$\times \left[\frac{(q + p_2)^2 + \eta_1^2}{q^2 - (p_2 + i\eta_1)^2}\right]^{i\xi_2}, \tag{12}$$

$$M_a^{11} = \bar{\gamma}_2^i \gamma_1^j J_1^{11}(p_a),\ \bar{\gamma}_2^2 = e_2[p_2 \nabla_{p_2} - (v_2 - \frac{i}{2}[\vec{\sigma}k_2])\frac{\partial}{\partial\eta_2}]|_{\eta_2 \to 0}, \tag{13}$$

$$J_1^{21}(p) = \frac{1}{ip^3}\int_1^\infty \frac{t^{i\xi}\,dt}{at^2 - 2bt + a^*}\{\frac{at^2 - 2bt + a^*}{a^1t^2 - 2b^1t + a^{1*}}\}^{i\xi_2}, \tag{14}$$

$$N_2^2 = 2\pi\xi_2/(1 - e^{-2\pi\xi_2}),\ \xi_2 = m\alpha Z/p_2,\ q = k_2 - k_1,$$

where a, b and a^1, b^1 are defined by formula (10), in which a and b should be replaced by $\vec{n}_2 = \vec{v}_2/p$, $\vec{v}_2 = \vec{k}_2 + \vec{p}_2$, ∇_{p2} in formula (13) does not act on $\vec{v}_2$ and $\vec{\xi}_2$, and for a^1 and b^1 we have $\vec{n}_2 = (\vec{v}_2 + \vec{p}_2)/p$, $i\mu_2 = (p_2 + i\eta_2)/p$, $\vec{p}_2$ - momentum of the emitted electron, and $\bar{\gamma}_2^{\ 1} = \bar{\gamma}_2^{\ 2}\ (\vec{k}_2 \to - \vec{k}_1)$.

The ionization cross section is determined by formula (10) multiplied by d^3p_2.

The authors are grateful to V. N. Efimov, A. I. Mikhailov, and L. P. Rappoport for a discussion.

[1] M. Gavrila, Phys. Rev. 163, 147 (1967).

[2] B. A. Zon, N. L. Manakov, and L. P. Rappoport, Zh. Eksp. Teor. Fiz. 55, 924 (1968) [Sov. Phys.-JETP 28 (1969)].

[3] V. G. Gorshkov, ibid. 47, 352 (1964) [20, 234 (1965)].

[4] C. Fronsdal, Preprint, Miramare-Trieste I c/68/85.

[5] V. G. Gorshkov, Zh. Eksp. Teor. Fiz. 41, 977 (1961) [Sov. Phys.-JETP 14, 694 (1962)].

NONLINEAR SCATTERING OF ION-ACOUSTIC OSCILLATIONS BY ELECTRONS

V. L. Sizonenko and K. N. Stepanov
Physico-technical Institute, USSR Academy of Sciences

Submitted 4 February 1969
ZhETF Pis. Red. 9, No. 8, 468-472 (20 April 1969)

The development of an ion-acoustic plasma instability when the electrons move relative to the ions is limited by the nonlinear interaction of these waves. In the present paper we consider the nonlinear scattering of ion-acoustic oscillations by electrons in a plasma situated in an external electric field of intensity much lower than the critical value ($E \ll E_{cr}$).

We shall show that the scattering of the ion-acoustic oscillations by the electrons is much stronger than by the ions, and estimate the turbulence level of the ion-acoustic pulsations and the turbulent friction force acting on the electrons (and ions) and responsible for the appearance of the anomalous reistance in the plasma, which can be of the order of and larger than the resistance due to the Coulomb collisions.