$$A_1 = 8r_0 N_2 \{ 2(M_a^{21} + M_b^{12}) + \frac{1}{2}\Gamma_1^1 \frac{\partial}{\partial\eta_1} \frac{(e_1 e_2)}{(q + p_2)^2 + \eta_1^2} \times$$

$$\times \left[\frac{(q + p_2)^2 + \eta_1^2}{q^2 - (p_2 + i\eta_1)^2}\right]^{i\xi_2}, \qquad (12)$$

$$M_a^{ll} = \bar{\gamma}_2^i \gamma_1^l J_1^{ll}(p_a), \quad \bar{\gamma}_2^2 = e_2[p_2 \nabla_{p2} - (v_2 - \frac{i}{2}[\vec{\sigma} k_2]) \frac{\partial}{\partial \eta_2}]|_{\eta_2 \to 0}, \qquad (13)$$

$$J_1^{21}(p) = \frac{1}{ip^3} \int_1^\infty \frac{t^{i\xi} dt}{at^2 - 2bt + a^*} \{ \frac{at^2 - 2bt + a^*}{a^1 t^2 - 2b^1 t + a^{1*}} \}^{i\xi_2}, \qquad (14)$$

$$N_2^2 = 2\pi\xi_2/(1 - e^{-2\pi\xi_2}), \quad \xi_2 = m\alpha Z/p_2, \quad q = k_2 - k_1,$$

where a, b and a^1, b^1 are defined by formula (10), in which a and b should be replaced by $\vec{n}_2 = \vec{v}_2/p$, $\vec{v}_2 = \vec{k}_2 + \vec{p}_2$, ∇_{p2} in formula (13) does not act on $\vec{v}_2$ and $\vec{\xi}_2$, and for a^1 and b^1 we have $\vec{n}_2 = (\vec{v}_2 + \vec{p}_2)/p$, $i\mu_2 = (p_2 + i\eta_2)/p$, $\vec{p}_2$ - momentum of the emitted electron, and $\bar{\gamma}_2^{\,1} = \bar{\gamma}_2^{\,2}$ $(\vec{k}_2 \to - \vec{k}_1)$.

The ionization cross section is determined by formula (10) multiplied by d^3p_2.

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NONLINEAR SCATTERING OF ION-ACOUSTIC OSCILLATIONS BY ELECTRONS

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The development of an ion-acoustic plasma instability when the electrons move relative to the ions is limited by the nonlinear interaction of these waves. In the present paper we consider the nonlinear scattering of ion-acoustic oscillations by electrons in a plasma situated in an external electric field of intensity much lower than the critical value $(E \ll E_{cr})$.

We shall show that the scattering of the ion-acoustic oscillations by the electrons is much stronger than by the ions, and estimate the turbulence level of the ion-acoustic pulsations and the turbulent friction force acting on the electrons (and ions) and responsible for the appearance of the anomalous reistance in the plasma, which can be of the order of and larger than the resistance due to the Coulomb collisions.

The nonlinear dispersion equation for the intensity $I_k = |\phi_k|^2$ of the ion-acoustic oscillations is given by [1]

$$\epsilon(\mathbf{k},\omega) I_k = \frac{1}{(2\pi)^3} \int d\mathbf{k}' U_{\mathbf{k},\mathbf{k}'} I_{k'} I_k , \tag{1}$$

where

$$U_{\mathbf{k},\mathbf{k}'} = \frac{4\pi e^4 (\mathbf{k}\mathbf{k}')}{m_e^3 k^2} \{ [(\mathbf{k}\mathbf{k}'') \frac{\partial^2}{\partial\omega\,\partial\omega''} - k^2 \frac{\partial^2}{\partial\omega^2}] \times$$

$$\times \int \frac{(\mathbf{k}'\partial f_0/\partial \mathbf{v})\,d\mathbf{v}}{(\omega-\mathbf{k}\mathbf{v})(\omega'-\mathbf{k}'\mathbf{v})(\omega''-\mathbf{k}''\mathbf{v})} - [\frac{1}{3}(\mathbf{k}\mathbf{k}')\frac{\partial^3}{\partial\omega^3} + \frac{1}{2}k^2 \frac{\partial^3}{\partial\omega^2\partial\omega''}] \times$$

$$\times \int \frac{(\mathbf{k}\,\partial f_0/\partial \mathbf{v})\,\partial\mathbf{v}}{(\omega-\mathbf{k}\mathbf{v})(\omega''-\mathbf{k}''\mathbf{v})} \}, \tag{2}$$

$$\omega = \omega_k + i0, \quad \omega' = \omega_k - i0, \quad \omega'' = \omega_k - \omega_{k'} + i0, \quad \mathbf{k}'' = \mathbf{k} - \mathbf{k}',$$

$$\omega_k = \frac{k v_s}{(1+k^2 r_d^2)^{1/2}}, \quad v_s = \left(\frac{T_e}{m_i}\right)^{1/2}, \quad r_d = \left(\frac{T_e}{4\pi e^2 n_0}\right)^{1/2} = \frac{v_{Te}}{\omega_{pe}} .$$

Since the ion-acoustic oscillations have a non-decaying spectrum, we have discarded the decay terms in the right side of (1), and also the term proportional to $v_{k,k''} v_{k'',k}$ (see [1]), which is our case of low current velocity u is smaller than $U_{k,k'}$ by a factor $v_{Te}/u \gg 1$.

It is necessary to add to (1) the kinetic equations for the averaged electron distribution function $f_0(\vec{v}, t)$ with allowance for the collision integral and the scattering of the electrons by the waves.

Let us estimate the value of $U_{k,k'}$, assuming for simplicity that f_0 has the form of a Maxwellian distribution shifted by an amount u. If $u \sim v_s$, $k \sim k'$, and the angle θ between $\vec{k}$ and $\vec{k}'$ is not close to zero ($\theta \sim 1$), then $U_{k,k'}$ is smaller by a factor $(m_i/m_u)^2$ than the quantity $U_{k,k'}$ at $\theta \leq (m_e/m_i)^{1/2}$, $k \sim k' \sim 1/r_d$, and $u \geq v_s$ we obtain in order of magnitude[1)]

$$U_{\mathbf{k},\mathbf{k}'} \sim i e^2 m_i v_s u k^2 r_d^2 / (m_e v_{Te} v_s)^3. \tag{3}$$

Since

$$\epsilon I_k \approx \frac{i}{2}\frac{\partial\epsilon}{\partial\omega}\left(\frac{\partial I_k}{\partial t} - 2\gamma_l I_k\right),$$

where $\gamma_\ell = \gamma_e - \gamma_i$ is the linear growth increment, γ_e the electron growth increment, and γ_i the ion damping decrement, we obtain from (1) for the steady state

$$i\gamma_l \partial\epsilon/\partial\omega + \int U_{\mathbf{k},\mathbf{k}'} I_{k'} d\mathbf{k}' = 0.$$

1) The singularities are eliminated from the integrals with respect to the velocities (2) by the method described in [11].

We thus get, using (3), that the energy of the ion-sound oscillations equals

$$W = \frac{1}{8\pi} \int \omega \frac{\partial \epsilon}{\partial \omega} k^2 I_k dk \sim \frac{m_e}{m_i} n_0 T_e \quad (u \gtrsim v_s). \tag{4}$$

The usually accounted for scattering by ions [1 - 3] yields a much larger value for W:

$$W/n_0 T_e \sim (u/v_{Te})(T_e/T_i). \tag{5}$$

This means that the principal role in the limitation of the growth of the ion-sound instability is played by the scattering of the oscillations by the electrons, and the scattering by the ions can be neglected.

The current velocity of the electrons is determined from the equations of motion

$$\frac{\partial u}{\partial t} = \frac{e}{m_e} E - (\nu_{col} + \nu_{turb}) u, \tag{6}$$

where ν_{col} is a quantity on the order of the electron-ion collision frequency, and $m_e \nu_{turb} u$ is the friction force exerted on the electrons by the ion-acoustic oscillations,

$$\nu_{turb} = \frac{e^2}{4\pi^3 m_e^2 v_{Te}^2 u^2 v_s} \int dk \frac{\mathbf{k u}}{k} (1 + k^2 r_d^2)^{3/2} \gamma_e I_k.$$

In order of magnitude, with allowance for (4), we get

$$\nu_{turb} = a(m_e/m_i)\,\omega_{pe} \qquad (a \sim 1). \tag{7}$$

The scattering of the electrons by the ion-acoustic oscillations will lead to a distortion of the electron distribution function, and in particular to their heating [7]. In the considered case of three-dimensional oscillations, the function f_0 changes in a wide interval of velocities $\Delta v_{x,y,z} \sim v_{Te}$. The heating time $\tau = v^2_{Te}/D$ (where D is the coefficient of quasilinear diffusion in velocity space) turns out to be, when $\nu_{col} < \nu_{turb}$ and allowance is made for formula (4), of the order of $\tau \sim v^2_{Te}/u^2\nu_{turb}$.

The appearance of anomalous plasma resistance than $u < v_{Te}$, which was first observed in a linear gas discharge [8] and recently in toroidal discharges [4 - 6, 10] can be related to the development of ion-acoustic instability.

The experimental data [4, 8, 10] agree with the foregoing estimates. For example, in experiments with a helium discharge in the "Sirius" stellarator we have H_0 = 10 kOe, $n_0 = 4 \times 10^{13}$ cm^{-3}, j = 100 A/cm^2, T_e = 104 eV (the temperature was determined from the diamagnetic signal), $E/E_{cr} = 0.06$, and $\nu_{exp}/\nu_{col} \sim 5$, from which we get $u = 1.5 \times 10^7$ cm/sec, $v_s = 6 \times 10^6$ cm/sec, $\nu_{col} \sim 2 \times 10^6$ sec^{-1}, and $\nu_{turb} \sim 2 \times 10^7$ sec^{-1}, so that $\nu_{turb}/\nu_{col} \sim 10$, and the plasma can be regarded as non-magnetized ($\omega^2_{pe}/\omega^2_{He} = 4$).

In the experiments of [5, 6] $\omega_{pe} < \omega_{He}$, and therefore the estimates obtained above for W and ν_{turb} call for refinement. In a strongly magnetized plasma ($\omega_{pe} \ll \omega_{He}$, $k^* v_{Te} \ll \omega_{He}$, where k^* is the maximum value of the wave vector at which the oscillations become stable), the electron motion is one-dimensional. In this case the matrix element $U_{k,k'}$ is of the order of magnitude of (3) even when $\theta \sim 1$. Therefore the noise level and the magnitude of the turbulent friction force decrease by a factor m_i/m_e compared with formulas (4) and (7) so that $\nu_{turb} \sim (m_e/m_i)^2\omega_{pe}$. In this case, however, it is necessary, when $u \sim v_s$, to take into account the formation of a "plateau" on the distribution function [9].

In a moderately magnetized plasma ($\omega_{pe} \sim \omega_{He}$), ν_{turb} should lie in the range $(m_e/m_i)^2\omega_{pe} < \nu_{turb} < (m_e/m_i)\omega_{pe}$. The effective collision frequency ν_{exp} calculated from measurements of the conductivity for the data on the T-3 "Tokamak" [6] and the C-stellarator [5] lie within this range.

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FEATURES OF PHOTON ECHO IN A GAS IN THE PRESENCE OF A MAGNETIC FIELD

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In both solids [1, 2] and gases [3], photon echo is customarily used to determine the relaxation time from the damping of the photon echo amplitude. Yet an investigation of the polarization effects of the photon echo can yield additional information concerning the physical processes in a medium. These polarization effects are particularly clearly pronounced in the presence of a magnetic field.

Let us consider photon echo in a gas situated in an external magnetic field $\vec{H}$. For concreteness, we confine ourselves to the quantum mechanical transition $J_2 = 1 \rightarrow J_1 = 0$ of a molecule with total angular momenta 1 and 0 of the upper and lower levels respectively. The passage of two successive light pulses and the formation of the photon echo are described by the well known Maxwell's equations and the quantum-mechanical density matrix. The final solution of the problem reduces to the following. During the time of passage of the short pulse, the influence of $\vec{H}$ can be neglected. The first light pulse polarizes the medium, directing the polarization current $\vec{j}(\vec{v})$ along the vector of the electric field intensity of