As is well known [4], an experimental determination of the gyromagnetic ratio by other methods is sometimes very difficult or impossible at all.

The angle ϕ_F of the rotation of the polarization of the electromagnetic wave on the path z as a result of the Faraday rotation in the gas equals, in order of magnitude, $\phi_F = N\lambda^2\Omega\gamma T_0^2 z$, where N and $1/\gamma$ are the density and the radiative lifetime of the active atoms, $1/T_0$ is the Doppler width of the level, and λ is the wavelength. For the parameters of the experiment of [3] we have $\phi_F \ll \phi$. In the general case it is necessary to add the contribution due to the Faraday rotation to the photon-echo polarization rotation angle ϕ obtained above.

As the result of the Faraday rotation, the polarizations of the transmitted light pulses and of the photon-echo will turn through the same angle ϕ_F, whereas the total angle of rotation of the photon-echo will equal the sum $\phi + \phi_F$. Therefore the separate contributions of each of the foregoing rotations can be easily determined experimentally.

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MAGNETIC-COULOMB LEVELS NEAR THE SADDLE POINTS

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The hypothesis that metastable Coulomb levels arise near the critical energy saddle points $E(\vec{k})$ was advanced [1] in connection with the interpretation of the structure of the intrinsic-absorption spectra of a number of crystals. Its theoretical justification, however, encounters serious difficulties [2, 3], and the very occurrence of such resonances on the absorption curve can be proved apparently only under special assumptions (for example, at a large mass ratio [3] or for deep levels as a result of the finite volume of the Brillouin zone [4]).

We show below that electron Coulomb levels electrons should arise in strong magnetic fields near all the critical points (in an attractive field for the minima and type-M_2 crests, and in a repulsive field), for M_1 crests and maxima we determine the angular dependence of the parameters of the resultant states.

Assume that in the original rectangular coordinates (x, y, z) the effective masses are $m_x = m_y = m_\perp$ and $m_z = m_\parallel$, with the signs of $m_\perp$ and $m_\parallel$ arbitrary, that the magnetic field $\vec{H}$ is given by $H_x = 0$ and $H_y/H_z = \tan\theta$, and that the vector potential is $\vec{A} = (1/2)\vec{H} \times \vec{r}$. Then, in the oblique system (ξ,η,ζ) in which the ζ axis is chosen along $\vec{H}$ and the same scale is retained (the Jacobian of the conversion is equal to unity), we have

$$\xi = \left|\frac{m_\perp}{M_\perp}\right|^{1/2} x, \quad \eta = \left|\frac{M_\perp}{m_\perp}\right|^{1/2}(\cos\theta\, y - \sin\theta\, z),$$

$$\zeta = \frac{M_\perp^2}{m_\perp}\left(\frac{\sin\theta}{m_\parallel} y + \frac{\cos\theta}{m_\perp} z\right),$$

$$\frac{1}{M_\perp(\theta)} = \left(\frac{\sin^2\theta}{m_\parallel m_\perp} + \frac{\cos^2\theta}{m_\perp^2}\right)^{1/2}, \quad M_\parallel(\theta) = m_\parallel\left(\frac{m_\perp}{M_\perp(\theta)}\right)^2, \tag{1}$$

the Hamiltonian is

$$\mathcal{H} = \frac{\operatorname{sign} m_\perp}{2M_\perp}\left[\left(p_\xi - \frac{eH}{2c}\eta\right)^2 + \left(p_\eta + \frac{eH}{2c}\xi\right)^2\right] - $$

$$- \frac{\hbar^2}{2M_\parallel}\frac{\partial^2}{\partial\zeta^2} - \frac{Ze^2}{\kappa r}, \tag{2}$$

where Z is the charge of the center and

$$r^2 = \zeta^2 + \left|\frac{M_\perp}{m_\perp}\right|^{3/2} \frac{m_\parallel - m_\perp}{m_\parallel} \sin 2\theta \sin\phi \; \rho\zeta +$$

$$+ \left|\frac{M_\perp}{m_\perp}\right| \left\{\cos^2\phi + \left(\frac{M_\perp}{m_\perp m_\parallel}\right)^2 (m_\parallel^2 \cos^2\theta + m_\perp^2 \sin^2\theta) \sin^2\phi\right\} \rho^2 ; \tag{3}$$

here ρ and ϕ are polar coordinates in the (ξ, η) plane.

As follows from (1) and (2), Landau quantization occurs when $m_\parallel/m_\perp > 0$, and also when $m_\parallel/m_\perp < 0$ if $\theta < \theta_{cr}$ and $\tan^2\theta_{cr} = |m_\parallel/m_\perp|$. This correspondsto trajectories located near the critical point in the k-space [5, 6]. When $\theta > \theta_{cr}$, the quasi-classical orbit in the magnetic field extends over a distance on the order of the dimensions of the Brillouin zone [5, 7], and the effective-mass method is not applicable; we exclude this case from considerations. When $\theta \to \theta_{cr}$ we get $M_\perp \to \infty$ and $M \to 0$.

If the Landau energy $\hbar\omega(\theta)$ ($\omega(\theta) = eH/M_\perp c$) is much larger than the Coulomb energy, the spectrum of the Hamiltonian (2) - (3) can be obtained in the adiabatic approximation in analogy with [8, 9]. The Landau levels are located above the critical point when $m_\perp > 0$, and below the critical point when $m_\perp < 0$; the first level corresponds to the states $n_p = 0$, $m \leq 0$ [10]. It is obvious from (2) that Coulomb levels can arise near them only if $Zm_\perp > 0$. The Coulomb level lies below the corresponding Landau level when $Z > 0$ (attractive potential) and above it when $Z < 0$ (repulsion).

In the lowest approximation, the wave function $F(\zeta)$ and the energy E_c of the ground Coulomb level are equal to

$$F(\zeta) \approx \frac{1}{\sqrt{na}} W_{n,1/2}\left(\frac{2\zeta}{na}\right), \quad E_c \approx -\frac{\hbar^2}{2M_\parallel (na)^2}, \tag{4}$$

where $W_{n,\,1/2}$ is the Whitaker function ($W_{n,\,1/2}(0) = 1$) and

$$a_0 = \frac{\kappa \hbar^2}{Z m_{\parallel} e^2}, \quad a = \frac{\kappa \hbar^2}{Z M_{\parallel} e^2}, \quad \frac{1}{n} = \ln\left(\frac{a\sqrt{a a_0}}{4\lambda^2}\right), \tag{5}$$

$$\lambda^2 = \frac{c\hbar}{eH}.$$

It is seen from (4) that na plays the role of the radius of the longitudinal motion. Since $a \to \infty$ as $\theta \to \theta_{cr}$, we take the difference between a and a_0 under the logarithm sign in (5).

Since, according to (3), the Coulomb potential has no axial symmetry in the new coordinates, energy matrix elements that are both diagonal and non-diagonal in m arise in the next higher approximation; they are of the order of nE_c. Therefore the splitting of the Coulomb levels coming from bands with different m is on the order of nE_c.

The magnetic-Coulomb level arising near the M_1 and M_2 crests lie in the continuous spectrum of the bands with larger n_ρ. This produces decays of the bound states, accompanied by a transition of the electron into states with larger n_ρ and with momenta k_ζ determined by energy conservation. The matrix element of the decay of the bound state with $n_\rho = 0$ at $\theta = 0$ (when m is conserved) equals

$$\langle 0m | \frac{Ze^2}{\kappa r} | n_\rho m k_\zeta \rangle = \frac{Ze^2}{\kappa} \frac{1}{\sqrt{2\pi n a_0}} \left(\frac{|m|!}{n_\rho!\,(n_\rho + |m|)!}\right)^{1/2} \times \\ \times \int_0^\infty d\sigma \frac{e^{-\sigma}\sigma^{n_\rho + |m|}}{(\sigma + |m_\parallel/m_\perp|\, n_\rho)^{|m|+1}}; \tag{6}$$

The final-state function is assumed to be normalized to the interval k_ζ. According to (6), it decreases with increasing m, and we present the estimate of the total decay probability only for m = 0; the series converges like $n_\rho^{-5/2}$, and [1)]

$$\frac{\Gamma_{00}}{E_c} < 4Z^2 n \frac{\lambda}{a_0} \left|\frac{m_\perp}{m_\parallel}\right|^{1/2} \left[\min\left(1, \left|\frac{m_\perp}{m_\parallel}\right|\right)\right]^2. \tag{7}$$

In the assumed approximation, $n\lambda/a_0 \ll 1$ and consequently $\Gamma_{00} \ll E_c$. A rough estimate of the probability of a decay with large momentum transfer, under the influence of a potential of atomic order acting at distances on the order of the lattice constant d, yields $\Gamma/E_c \approx n\kappa(d/\lambda)^2$, which is small under the conditions when the effective-mass method is applicable. Consequently, the criterion for the existence of metastable states is satisfied.

One of the most interesting cases, in which the appearance of magnetic-Coulomb levels can be expected near the saddle points, is the impurity absorption of light by electrons of deep impurity levels. In multiply-charged centers, the electron excited by the light in the

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[1)] A similar formula was obtained in [11] for the width of the excited levels of a local center.

conduction band turns out to be in the repulsive field of the negatively-charged center, and should therefore have metastable levels near M_1 in a strong magnetic field. The total oscillator strength for the transition to the impurity levels with definite n_ρ is of the order of

$$f_{n_\rho} \sim d^3 \, |\psi(0)|^2 = \frac{1}{2\pi} \frac{d^3}{\lambda^2 a n}, \tag{8}$$

and depends strongly on the magnitude and direction of $\vec{H}$. In those cases when these transitions fall in the transparency zone, they apparently should be observable.

If the transitions arise between states of large radius at parabolic and hyperbolic points, then the order of magnitude of the oscillator strength can reach unity, and in the limit of large h it depends weakly (logarithmically) on the magnitude of the field and strongly ($\sim M_\perp^{-3/2}(\theta)$) on its orientation.

The foregoing results are valid when $E_c \ll \hbar\omega$; contributing to this criterion, as usual, are large values of κ and small values of $m_\perp$ and $m_\parallel$. The case of small m is apparently encountered most rarely for saddle points; however, as seen from (7), at small values of $|m_\perp/m_\parallel|$ the level width Γ is small even when $E_c/\hbar\omega$ is not very small, and therefore the main qualitative conclusions should remain in force. We note incidently that satisfaction of the adiabaticity condition is facilitated when $\theta \to \theta_{cr}$.

A similar situation takes place also for hyperbolic excitons, but the theory for them is more complicated; it will be developed separately.

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