ELECTROSONIC WAVES IN A PLASMA WITH NEGATIVE DIELECTRIC CONSTANT

V. I. Karpman
Institute of Terrestrial Magnetism, Ionosphere, and Radiowave Propagation, USSR Academy of Sciences
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We consider in this paper the nonlinear mechanism of penetration of a transverse electromagnetic field in a plasma with negative dielectric constant

$$\epsilon = 1 - \omega_0^2/\omega^2, \quad \omega_0^2 = 4\pi nc^2/m_c. \tag{1}$$

According to the linear theory, an electromagnetic wave with frequency lower than the plasma frequency ω_0 can penetrate in such a plasma only to a depth on the order of the skin layer μ^{-1}, where

$$\mu^2 = (\omega_0^2 - \omega^2)/c^2.$$

The nonlinear change of the plasma density under the influence of the radiation pressure forces leads to the possible propagation in the plasma of density-rarefaction waves in which an electromagnetic field with $\omega < \omega_0$ is locked-in; such waves will be called electrosonic [1].

The main equations for electrosonic waves of small (but finite) amplitude E near the threshold ($\omega_0 - \omega << \omega_0$) are of the form [1]

$$2i\frac{\partial E}{\partial t} + \frac{c^2}{\omega_i}\frac{\partial^2 E}{\partial x^2} - \omega_0(\nu+\gamma^2)E(x,t) = 0, \tag{2}$$

$$\frac{\partial^2 \nu(x,t)}{\partial t^2} - c_s^2\frac{\partial^2 \nu}{\partial x^2} = c_s^2\frac{\partial^2}{\partial x^2}\left(\frac{|E|^2}{E_c^2}\right), \tag{3}$$

where the electric field intensity $\mathcal{E}$ is connected with the complex amplitude by the relation $\mathcal{E} = \mathrm{Re}\{E(x,t)\exp(-i\omega t)\}$ and

$$\nu = \frac{\rho - \rho_0}{\rho_0}, \quad \gamma^2 = \frac{\omega_0^2 - \omega^2}{\omega_0^2}, \quad E_c^2 = 16\pi\rho_0 c_s^2, \tag{4}$$

$\rho(x, t)$ is the plasma density, ρ_0 the unperturbed density, and $c_s = (T_e/m_i)^{1/2}$ is the velocity of ion sound in the plasma, which is assumed to be strongly non-isothermal ($T_e >> T_i$). It is also assumed that

$$\nu << 1, \quad |E|^2 << \gamma^2 E_c^2, \quad c_s/c << \gamma. \tag{5}$$

Under these conditions we obtain the following results.

1. If an electromagnetic wave

$$\mathcal{E}_e = E_e(t)\cos\omega(t - \frac{x}{c}), \tag{6}$$

is incident on the boundary of the plasma (which we conveniently locate at $x = -\infty$) within a finite time interval $t_1 < t < t_2$, then the field in the plasma at the plasma boundary is given by

$$E(-\infty,\ t) \approx 2E_e(t), \tag{7}$$

and the total energy remaining in the plasma after the removal of the external field ($t > t_2$) is

$$W \approx \frac{c_s}{8\pi} \int_{t_1}^{t_2} dt\, E^2(-\infty,\ t) \approx \frac{c_s}{2\pi} \int_{t_1}^{t_2} dt\, E_e^2(t). \tag{8}$$

The phase factor in the expression for the field $\mathcal{E}(x,\ t)$ can always choose such that the amplitude $E(x,\ t)$ is real, accurate to terms of order $c_s/\gamma c$.

The energy density of the electrosonic waves and the plasma, according to formula (1.14) of [1], is $E^2(x,\ t)/8\pi$ if conditions (5) are satisfied). It therefore follows from (8) that the energy flux into the plasma equals (approximately) the energy density on the boundary, multiplied by the speed of sound c_s (i.e., the velocity of the electrosonic waves under the conditions (5) is close to the velocity of the sound).

2. We consider now the evolution of the electroacoustic wave produced in the plasma when $t > t_2$. Let the profile of the amplitude of the electric field in the electroacoustic wave at $t = t_0 >> t_2$ be of the form

$$E(x,\ t) = f(x). \tag{9}$$

It turns out here that the following relations hold

$$f(x) \approx \text{const}\ e^{-\mu|x|} \qquad (x \to \pm\infty), \tag{10}$$

$$\frac{1}{8\pi} \int_{-\infty}^{\infty} f^2(x)dx = \frac{c_s}{2\pi} \int_{t_1}^{t_2} dt\, E_e^2(t)\, dt\ . \tag{11}$$

The approximate solution of Eqs. (3) (under conditions (5)) is given by

$$E^2(x,t) = E_m^2\ \frac{dr(\xi)}{d\xi}\ \text{sech}^2\{\mu[\frac{E_m^2 c_s}{4\gamma^2 E_c^2} t + r(\xi)]\}, \tag{12}$$

$$\nu = -\gamma^2\, \text{sech}^2\{\mu \frac{E_m^2 c_s}{4\gamma^2 E_c^2} t + r(\xi)]\}, \tag{13}$$

$$r(\xi) = \frac{1}{2\mu} \ln\left[\frac{2E_m^2 - \mu \int_\xi^\infty f^2(\eta)d\eta}{\mu \int_\xi^\infty f^2(\eta)d\eta}\right] - \frac{E_m^2 c_s}{4\gamma^2 E_c^2} t_0, \tag{14}$$

$$\xi = x - c_s t,$$

$$E_m^2 = 2\mu c_s \int_{t_1}^{t_2} E_e^2(t)\,dt\,. \tag{15}$$

The function $\tau(\xi)$ increases monotonically with increasing ξ and, according to formula (10), has the following asymptotic form for large ξ:

$$\tau(\xi) \approx \xi, \qquad (\xi \to \pm\infty)\,. \tag{16}$$

It follows from (16) and (12) that at large t the electrosonic wave is described by the following limiting relations

$$E(x,t) \to E_m \operatorname{sech}[\mu(x - Mc_s t)]\,, \qquad (t \to \infty) \tag{17}$$

$$\nu(x,t) \to -\gamma^2 \operatorname{sech}^2[\mu(x - Mc_s t)]\,,$$

$$M = 1 - \frac{E_m^2}{4\gamma^2 E_c^2} \tag{18}$$

The right sides of (17) describe, in accordance with [1], an electrosonic soliton with amplitude E_m and a Mach number M; owing to the conditions of formula (5), M should be close to unity. Thus expression (18) is equivalent to the general relation between the Mach number and the soliton amplitude, obtained in [1] (see formula (3.18) of that paper).

Thus, the radiation (6) incident on the plasma forms in the latter an electrosonic wave which, at sufficiently large t, is transformed into a soliton with an amplitude given by (15).

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[1] V. Ts. Gurovich and V. I. Karpman, Zh. Eksp. Teor. Fiz. 56, No. 5, (1969) [Sov. Phys. JETP 29, No. 5 (1969)].

E R R A T A

The reference following the article by V. I. Karpman (Vol. 9, No. 8), on p. 293, should read "... Zh. Eksp. Teor. Fiz. 56, 1952 (1969)."

The footnote of the article by L. D. Derkacheva and A. I. Krymova (Vol. 9, No. 10), on p. 345, should read "Appearance of one spectral line," not"Suppression of one spectral line."