$$E_m^2 = 2\mu c_s \int_{t_1}^{t_2} E_e^2(t)\,dt \,. \tag{15}$$

The function $\tau(\xi)$ increases monotonically with increasing ξ and, according to formula (10), has the following asymptotic form for large ξ:

$$\tau(\xi) \approx \xi, \qquad (\xi \to \pm\infty) \,. \tag{16}$$

It follows from (16) and (12) that at large t the electrosonic wave is described by the following limiting relations

$$E(x,t) \to E_m \operatorname{sech}[\mu(x - Mc_s t)], \qquad (t \to \infty) \tag{17}$$

$$\nu(x,t) \to -\gamma^2 \operatorname{sech}^2[\mu(x - Mc_s t)],$$

$$M = 1 - \frac{E_m^2}{4\gamma^2 E_c^2} \tag{18}$$

The right sides of (17) describe, in accordance with [1], an electrosonic soliton with amplitude E_m and a Mach number M; owing to the conditions of formula (5), M should be close to unity. Thus expression (18) is equivalent to the general relation between the Mach number and the soliton amplitude, obtained in [1] (see formula (3.18) of that paper).

Thus, the radiation (6) incident on the plasma forms in the latter an electrosonic wave which, at sufficiently large t, is transformed into a soliton with an amplitude given by (15).

I take the opportunity to thank V. P. Sokolov for useful discussions of the questions considered.

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POSSIBILITY OF GALAXY FORMATION IN THE LEMAITRE MODEL

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A classical analysis of perturbations in an infinite gravitating medium with a constant density ε and velocity of sound $u = \sqrt{dp/d\varepsilon}$ leads to the Jeans criterion, namely, the perturbations that grow exponentially are those with wave numbers smaller than the limiting (Jeans) value

$$k_j^2 = \frac{\kappa\epsilon}{2u^2} \tag{1}$$

(κ is Einstein's gravitational constant, c = 1 is the velocity of light).

The relativistic analysis of the open and closed models of the universe, made by E. Lifshitz [1], shows that in the case of expansion the perturbations increase no faster than in a power-law fashion. However, as shown by the author [2], in the Lemaitre model with a cosmo logic constant Λ close to the critical value $\Lambda_c = \sigma_c^{-2}$, near the critical radius $a_c = \sqrt{2/\kappa\varepsilon_c}$ (on the plateau), the perturbations increase quite rapidly, and their total growth is proportional to $\Delta^{-1} = \Lambda_c/\Lambda - \Lambda_c >> 1$.

We shall show now that in this case the Jeans criterion is also satisfied. We assume that the speed of sound is constant on the plateau and is much smaller than the speed of light, $u << 1$, and we are interested in scalar perturbations whose dimension is $\ell \sim a_c u$, corresponding to $n >> 1$ (the number of the harmonics in the expansion of the perturbations in the 4-spherical functions $q^{(n)}$). The equations for the Lifshitz functions ξ and ζ will then be [2]:

$$\xi' + \xi\left(\frac{P'}{P} - \frac{2}{a}\right) = \frac{u^2\zeta}{2\sqrt{P}}, \qquad \zeta' + \frac{\zeta}{a} = -\frac{2n^2\xi}{\sqrt{P}}. \tag{2}$$

The prime denotes here differentiation with respect to the radius of curvature a;

$$P(a) = \frac{2}{3}a_c a - a^2 + \frac{\Lambda a^4}{3}.$$

The density perturbations are expressed in terms of the solutions of (2)

$$\frac{\delta\epsilon}{\epsilon} = -\frac{Q^{(n)}\sqrt{P}}{4a_c a}\zeta. \tag{3}$$

Eliminating the function ξ from (2) and going over to the variable

$$\eta = \int\frac{da}{\sqrt{P}},$$

we obtain the equation

$$\ddot{\zeta} + \dot{\zeta}[2(\ln\dot{a})^{\cdot} - (\ln a)^{\cdot}] + \zeta[(nu)^2 + 3(\ln a)^{\cdot\cdot}] = 0. \tag{4}$$

The dot denotes differentiation with respect to η.

Near the critical radius $|a - a_c| << a$, the solution of this equation is of the form

$$\zeta = \text{const}\,\frac{1}{\dot{a}}\begin{cases}\exp\{\pm\eta i\sqrt{(nu)^2-1}\} & \text{if} \quad nu > 1\\ \exp\{\pm\eta\sqrt{1-(nu)^2}\} & \text{if} \quad nu < 1\end{cases}. \tag{5}$$

The corresponding density perturbations at $nu < 1$ will have an exponentally growing solution

$$\frac{\delta\epsilon}{\epsilon} = \text{const}\,Q^{(n)}\exp\{\eta\sqrt{1-(nu)^2}\}. \tag{6}$$

On going over to the wave vectors $k = n/a_c$, the criterion $un < 1$ coincides with the Jeans criterion (1). This was demonstrated by Bonnor [3] within the framework of Newtonian cosmology for the stationary Einstein model.

The total growth of the density perturbations on the plateau can be estimated by substituting $\eta \sim \ln\Delta^{-1}$, the time of passage through the plateau, which yields

$$K_n = \left[\frac{\delta\epsilon}{\epsilon}(a > a_c) \Big/ \frac{\delta\epsilon}{\epsilon}(a < a_c)\right] \approx \Delta^{-\sqrt{1-(nu)^2}}. \tag{7}$$

At $nu << 1$ this value coincides, apart from a coefficient, with the expression obtained in [2] for the equation of state p = 0. We assume now that the universe is described by a Lematire model with $\Delta \sim 10^{-4} - 10^{-5}$ (Kardashev's estimate [4]) and that the stars were produced before the galaxies.

Let us find the fluctuation of the stellar density prior to the formation of the galaxies. Immediately after the formation of the stars we can assume that the condition for the quasi-closure of the subsystems is satisfied; this yields $(\delta\varepsilon/\varepsilon)^{(0)} \sim 1/\sqrt{N}$. Here N is the number of stars in a fluctuation of a given scale. We are particularly interested in fluctuations occurring in volumes containing a galactic number of stars, $N \sim 10^{10} - 10^{11}$. This gives an estimate of the initial inhomogeneities of the density of the galactic scales. On passing through the plateau, these fluctuations increase by approximately Δ^{-1} times and thus become of the order of unity. We did not take into account here the growth in the pre-critical stage, which is linear in the radius of curvature, so that this estimate is satisfied with some margin.

The perturbations of the metric then also become of the order of unity, and the perturbations of the velocities are of the order of the speed of sound, which must be estimated from the random velocities of the stars, $u \sim 10^5 - 10^6$ cm/sec.

The dimension of the perturbations can be estimated from the Jeans criterion, putting $a_c \sim 3 \times 10^{27}$ cm

$$\ell \sim \frac{a_c}{n} \sim a_c u \sim 10^{23} \div 10^{24} \text{ cm},$$

which is in satisfactory agreement with the galactic scales. A certain overestimate of the dimension can be reasonably attributed to the fact that the speed of sound could increase noticeably during the nonlinear stage of galaxy formation.

Recognizing that the square of the initial fluctuations is inversely proportional to the cube of their dimension, we can obtain an estimating formula for the distribution of the growth of the perturbations:

$$\left(\frac{\delta\epsilon}{\epsilon}\right)_n = \left(\frac{\delta\epsilon}{\epsilon}\right)^{(0)} K_n \sim \sqrt{\frac{m}{M}}\, n^{3/2} \exp\{\eta\sqrt{1-(nu)^2}\} \tag{8}$$

(m - average mass of the stars, M - total mass of the universe).

This function has a maximum at n corresponding to galactic dimensions $(nu)_{max} \approx (3/2 \times \ln\Delta^{-1})^{1/2}$; its value at the maximum reaches an order of unity on the plateau. We note the very steep decrease to the right of the maximum. This circumstance should indicate an asymmetry in the dispersion of the galaxies by dimensions.

After $\delta\varepsilon/\varepsilon \sim 1$ is reached in a certain region of n, separation of the galaxies occurs and its investigation should take into account the nonlinearity of Einstein's equation, both the gravitational and hydrodynamic. In this stage, in view of the large dispersion of the density, the speed of sound depends on the coordinates, so that there is no single equation of state in the universe. However, after the separation of the galaxies, the universe can be regarded as a galactic gas with an equation of state $u_1 = \sqrt{dp_1/d\varepsilon}$, and the speed of sound corresponds to the random velocities of the galaxies, $u_1 \sim 10^7 - 10^8$ cm/sec. If the radius of curvature is still on the plateau at the same time, then galactic clusters with dimensions

$$\ell \sim a_c u_1 \sim 10^{25} \text{ cm.}$$

begin to grow.

We note that similar relations can be obtained also in ordinary models, assuming that the star production occurs at a sufficiently early stage.

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INFLUENCE OF NONMONOCHROMATIC PUMPING ON THE FORM OF THE SPECTRUM OF STIMULATED MANDEL'SHTAM-BRILLOUIN SCATTERING

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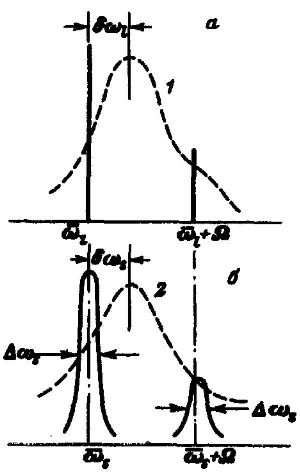
1. The influence of periodic modulation of the pumping on the spectrum of stimulated Mandel'shtam-Brillouin scattering (SMBS), insofar as we know, has not been investigated theoretically. This problem is of interest in connection with the use of SMBS for the measurement of the velocity v of hypersound by observing the shift $\bar{\omega}_p$ of the average frequency of the scattered light $\bar{\omega}_s = \bar{\omega}_\ell - \bar{\omega}_p$ relative to the average pump frequency $\bar{\omega}_\ell$: $v = \bar{\omega}_p c/2n\bar{\omega}_\ell$, where c/n is the velocity of light in the medium [1]. A relatively weak parasitic mode (PM) with frequency $\omega_\ell + \Omega, \Omega \ll \omega_p$ (Fig. a) disturbs the symmetry of the pump spectrum $G(\bar{\omega}_\ell + \omega)$, and also, generally speaking, of the SMBS spectrum $G(\bar{\omega}_s + \omega)$. The PM-induced average shifts of the maxima of these spectra, observed with a spectral instrument having an apparatus function $a(\omega)$, will be denoted by $\delta\omega_\ell$ and $\delta\omega_s$ (the observed spectra of the pump (1) and of the SMBS (2) are shown schematically (dashed) in Figs. a and b). The error in the estimate of the speed of sound amount in this case to



$$\frac{\delta v}{v} = \frac{\delta\omega_\ell - \delta\omega_s}{\bar{\omega}_p} \quad (1)$$

2. If the pump spectrum $G(\bar{\omega}_\ell + \omega)$ has a maximum at $\omega = 0$, then the maximum of the observed spectrum will be shifted by an amount

$$\delta\omega_\ell = \int_{-\infty}^{\infty} G(\bar{\omega}_\ell + \omega)\frac{\partial a(\omega)}{\partial\omega}d\omega \Big/ \int_{-\infty}^{\infty} G(\bar{\omega}_\ell + \omega)\frac{\partial^2 a(\omega)}{\partial\omega^2}d\omega. \quad (2)$$

An analogous expression can be written also for $\delta\omega_s$. Putting

$$G(\bar{\omega}_\ell + \omega) = \delta(\omega) + \mu^2\delta(\omega - \Omega),\ \mu^2 \ll 1, \quad (3)$$