of curvature is still on the plateau at the same time, then galactic clusters with dimensions

$$\ell \sim a_c u_1 \sim 10^{25} \text{ cm.}$$

begin to grow.

We note that similar relations can be obtained also in ordinary models, assuming that the star production occurs at a sufficiently early stage.

In conclusion, the author thanks I. M. Khalatnikov for fruitful discussions and N. S. Kardashev for a discussion of the observational data.

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# INFLUENCE OF NONMONOCHROMATIC PUMPING ON THE FORM OF THE SPECTRUM OF STIMULATED MANDEL'SHTAM-BRILLOUIN SCATTERING

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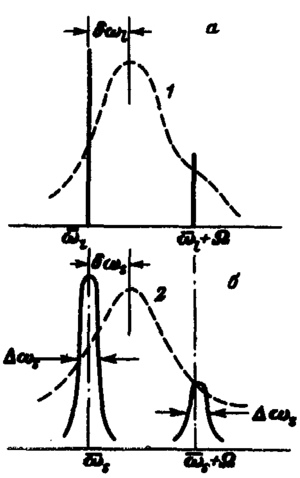
Submitted 5 March 1969
ZhETF Pis. Red. 9, No. 8, 487-490 (20 April 1969)

1. The influence of periodic modulation of the pumping on the spectrum of stimulated Mandel'shtam-Brillouin scattering (SMBS), insofar as we know, has not been investigated theoretically. This problem is of interest in connection with the use of SMBS for the measurement of the velocity v of hypersound by observing the shift $\bar{\omega}_p$ of the average frequency of the scattered light $\bar{\omega}_s = \bar{\omega}_\ell - \bar{\omega}_p$ relative to the average pump frequency $\bar{\omega}_\ell$: $v = \bar{\omega}_p c/2n\bar{\omega}_\ell$, where c/n is the velocity of light in the medium [1]. A relatively weak parasitic mode (PM) with frequency $\omega_\ell + \Omega, \Omega << \omega_p$ (Fig. a) disturbs the symmetry of the pump spectrum $G(\bar{\omega}_\ell + \omega)$, and also, generally speaking, of the SMBS spectrum $G(\bar{\omega}_s + \omega)$. The PM-induced average shifts of the maxima of these spectra, observed with a spectral instrument having an apparatus function $a(\omega)$, will be denoted by $\delta\omega_\ell$ and $\delta\omega_s$ (the observed spectra of the pump (1) and of the SMBS (2) are shown schematically (dashed) in Figs. a and b). The error in the estimate of the speed of sound amount in this case to




$$\frac{\delta v}{v} = \frac{\delta\omega_\ell - \delta\omega_s}{\bar{\omega}_p} \quad (1)$$

2. If the pump spectrum $G(\bar{\omega}_\ell + \omega)$ has a maximum at $\omega = 0$, then the maximum of the observed spectrum will be shifted by an amount

$$\delta\omega_\ell = \int_{-\infty}^{\infty} G(\bar{\omega}_\ell + \omega)\frac{\partial a(\omega)}{\partial\omega}d\omega \Big/ \int_{-\infty}^{\infty} G(\bar{\omega}_\ell + \omega)\frac{\partial^2 a(\omega)}{\partial\omega^2}d\omega. \quad (2)$$

An analogous expression can be written also for $\delta\omega_s$. Putting

$$G(\bar{\omega}_\ell + \omega) = \delta(\omega) + \mu^2\delta(\omega - \Omega), \ \mu^2 << 1, \quad (3)$$

and specifying the apparatus function in the form $a(\omega) = \exp(-\omega^2/h^2)$, we obtain

$$\delta\omega_\ell \simeq \mu^2 \Omega \exp(-\Omega^2/h^2). \tag{4}$$

3. In seeking to determine the SMBS spectrum $G(\bar{\omega}_s + \omega)$, we start from the general expression for the complex amplitude of the scattered light

$$E_s(t, x) \sim E_\ell(t) \int_0^\infty dt' \int_0^x dx' e^{-avt'} N(t-t', x-x') I_0 \times \times (2\sqrt{vg_s g_p x' \int_{t-t'}^{t} |E_\ell^2(\theta)| d\theta}), \tag{5}$$

where $E_\ell(t)$ is the pump amplitude, x the length of the scattering medium, $\alpha$ the sound-damping coefficient, $g_s = 1/4\ \bar{\beta}_s R_s \gamma$, $g_p = (1/16\pi)\gamma k_p$, $2\bar{k}_s = \bar{k}_p = \omega_p/v$, and $N(t, x)$ the $\delta$-correlated noise field, which produces the primary thermal pressure fluctuations that take part in the scattering [2];

$$\beta_s = -\left(\frac{1}{V}\frac{\partial V}{\partial p}\right)_s, \quad \gamma = \rho\left(\frac{\partial \epsilon}{\partial p}\right)_s \quad [1],$$

and $I_0$ is the Bessel function.

Formula (4) determines the solution of the simplified SMBS equations

$$\frac{\partial E_s}{\partial x} + ig_s E_\ell(t) P = 0; \quad \left(\frac{1}{v}\frac{\partial}{\partial t} + a\right)P + ig_p E_l^*(t) E_s = N(t, x)$$

which relate $E_s$, $E_\ell$ and N with the amplitude P of the pressure wave.

As follows from (5), the general expression for the SMBS spectrum, corresponding to an arbitrary periodic variation of $E_\ell(t)$, is

$$G(\bar{\omega}_s + \omega) \sim \int_0^x dx' \sum_n |d_n^2(\omega, x')|, \tag{6}$$

where the functions

$$d_n(\omega, x') = \int_0^\infty c_n(t', x') \exp\{-[av + i(\omega - \Omega n)]t'\} dt'$$

are expressed in terms of the Fourier components of the expansion

$$E_\ell(t) I_0(2\sqrt{vg_s g_p x' \int_{t-t'}^{t} |E_\ell^2(\theta)| d\theta}) = \sum_n c_n(x', t') e^{i\Omega nt}.$$

In monochromatic pumping ($E_\ell$ = const), in particular, formula (5) determines the distribution of the SMBS intensity over the frequencies

$$G(\bar{\omega}_s + \omega) \approx \exp \frac{2g_s g_p |E_\ell^2| x a^{-1}}{1 + (\omega/av)^2} - 1, \tag{7}$$

which coincides with that obtained in [3].

4. In the case of interest to us, the pump spectrum is determined by formula (3); the relative PM intensity is then equal to $\mu^2 \ll 1$, and we must put $E_\ell(t) = \bar{E}_\ell(1 + \mu e^{i\Omega t})$. The usual conditions for the observation of SMBS correspond to $\kappa = 2g_s g_p |E_\ell^2| x\alpha^{-1} \gg 1$. In addition, under typical conditions ($\alpha \approx 300$ cm$^{-1}$, $v = 10^5$ cm/sec, $\Omega = 2\pi 400 \times 10^{16}$ sec$^{-1}$, the

small parameter is $\alpha v/\Omega$. Allowance for these inequalities makes it possible to rewrite (5) in the form

$$G(\bar{\omega}_s + \omega) \approx e^{\kappa} \, [e^{-\omega^2/q^2} + \mu^2 e^{-(\omega-\Omega)^2/q^2}], \tag{8}$$

$$q = \frac{\alpha v}{\sqrt{\kappa}}, \quad \kappa >> 1, \quad \frac{\alpha v}{\Omega} << 1, \quad \mu << 1.$$

The obtained spectrum (8) consists of two narrow lines ($q << \Omega$ by virtue of the inequality $\alpha v << \Omega$) separated by the frequency $\Omega$ (Fig. b). Both lines have a Gaussian form and are characterized by the same width $\Delta_s = 2g\sqrt{\ln 2}$ (at half-intensity level). This same width is possessed also by the unperturbed spectrum (7) with which the first term of the sum (8) coincides, as can be readily verified, when $\kappa >> 1$.

Substituting (8) in place of $G(\bar{\omega}_\ell + \omega)$ in (2), we obtain the effective shift of the observed SMBS spectrum

$$\delta\omega_s \approx \mu^2 \Omega \frac{h^2}{h^2 + q^2} \exp\left(-\frac{\Omega^2}{h^2 + q^2}\right). \tag{9}$$

5. From formulas (1), (4), and (9) we can obtain an estimate for the error in the measurement of the velocity of hypersound, due to the single parasitic pump mode. For example, in the case of a "very poor" spectral instrument ($h^2 >> q^2$, $h^2 >> \Omega^2$) we have

$$\frac{\delta v}{v} \approx \frac{\mu^2 \Omega q^2}{h^2 \bar{\omega}_p} \approx \mu^2 10^{-2},$$

if we take $\Omega = 2\pi 400 \times 10^6$ sec$^{-1}$, $\bar{\omega}_p = 2\pi \times 6 \times 10^{19}$ sec$^{-1}$, and $q^2/h^2 = 0.1$.

6. The estimate obtained here for $\delta v/v$ is much lower than that indicated in [4]. The discrepancy between the results can be understood by recognizing that authors of [4] started from the assumption that the PM, the intensity of which is smaller than the threshold value, appears only in the pump spectrum, but does not "pass" to the SMBS spectrum. As seen from (3) and (8), under the conditions considered here the PM distorts the pump spectrum and the SMBS spectrum to an equal degree.

The author is grateful to I. L. Fabelinskii for a discussion of questions touched upon in the article.

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