ELECTRODYNAMICS OF THE "ELECTRONIC CRYSTAL"

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Submitted 11 March 1969
ZhETF Pis. Red. 9, No. 8, 491-494 (20 April 1969)

A number of papers (see, for example [1]) deal with the model of the so-called "electronic crystal," in which the electrons are in the field of a uniformly-diffuse positive charge. Account is taken here of the Coulomb interaction of the electrons, but the electrons as a whole move freely (there is no restoring force between the electrons and the positive core). The electron density is assumed to be sufficiently small to make the amplitude of the zero-point oscillations of the electron lattice much smaller than the period.

In [2, 3] they investigated the spectrum of the oscillations of such a crystal. It was found that in the limit of large wavelengths the oscillation spectrum contains two transverse branches with a linear dispersion law ($\omega = c_e k$) and a longitudinal branch with plasma frequency (ω_0). We wish to call attention to the fact that the spectrum of the transverse oscillations was found in these papers without allowance for the transverse electromagnetic fields (which are connected with these oscillations), and we shall show in what follows that allowance for these fields alters the spectrum at low frequencies noticeably.

For long waves we can confine ourselves to the continuous-medium approximation. We then obtain the following equations of motion and Maxwell's equations for the transverse displacement ξ and the electric field $\vec{E}$, in which the influence of the electrostatic forces is accounted for by introducing the transverse elasticity:

$$\frac{\partial^2 \xi}{\partial t^2} - c_e^2 \Delta \xi = \frac{eE}{m},$$
$$\Delta E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi n e}{c^2}\frac{\partial^2 \xi}{\partial t^2}, \quad (1)$$

where n is the electron density and c_e the velocity of the transverse "electron sound" [3].

From (1) we obtain for a plane wave $\exp(-i\omega t + ikx)$ the dispersion relation for the natural-oscillation frequencies

$$(\omega^2 - k^2c^2)(\omega^2 - k^2c_e^2) = \omega_0^2\omega^2. \quad (2)$$

Recognizing that $c \gg c_e$, we find that two branches of the dispersion equation (2):

$$\omega_1^2 = \omega_0^2 + k^2c^2,$$
$$\omega_2^2 = c^2c_e^2k^4 / (\omega_0^2 + k^2c^2). \quad (3)$$

We see that when $k \ll \omega_0/c$ the spectrum differs appreciably from the acoustic spectrum. The first branch corresponds to the propagation of transverse optical waves with frequency below the plasma frequency. The electrons in these oscillations behave almost like free electrons. The second branch, at small values of k, describes the low-frequency oscillations, the energy of which is contained mainly in the elastic energy and in the magnetic field. Only when $k \gg \omega_0/c$ do these oscillations go over into sound oscillations.

The change of the spectrum at low values of k ($\omega = cc_e k^2/\omega_0$) leads to a change of the thermodynamics of the electronic crystal at low temperatures (for example, the specific heat is proportional to $T^{3/2}$), but a more interesting fact, from the point of view, is that the crystal is transparent to the low-frequency electromagnetic waves ($\omega < \omega_0$) (the dielectric constant $\varepsilon > 0$). Indeed, for the medium under consideration $\varepsilon_\perp$ is given by

$$\epsilon_\perp(k, \omega) = \omega_0^2/\omega^2 - c_e^2 k^2. \tag{4}$$

At low frequencies $\omega \ll \omega_0$ we get from (4) and (3)

$$\epsilon_\perp(\omega) = \frac{c^2}{c_e^2}\,\frac{1 + \sqrt{1 + 4\dfrac{c_e^2}{c^2}\dfrac{\omega_0^2}{\omega^2}}}{2}. \tag{5}$$

We see from (5) that $\varepsilon_\perp(\omega) \gtrsim c^2/c_e^2 n$, and accordingly the transmission coefficient $D \lesssim c_e/c \ll 1$. The largest transmission coefficients correspond to frequencies $\omega \gtrsim \omega_0\,(c_e/c)$.

We have not taken into account anywhere above the dissipative processes. From among the possible damping mechanism, we take notice of the following two. It is possible that the damping corresponds to a certain effective viscosity. In this case it is necessary to add the term $\nu\Delta(\partial\xi/\partial t)$ in the right side of the equations of motion (1), and then at low frequencies we have

$$\omega_2 \cong \pm\frac{c\,c_e k^2}{\omega_0} - i\nu\frac{c^2 k^4}{2\omega_0^2}$$

and the damping at low frequencies is small. It is proportional to the square of the frequency.

On the other hand, if the dissipative processes are described by usual friction, then it is necessary to add the term $\tau^{-1}\partial\xi/\partial t$ in the right side of the equation of motion (1). In this case

$$\omega_2 = \frac{c_e c k^2}{\sqrt{\omega_0^2 + c^2 k^2}}\left[\pm\sqrt{1 - \lambda^2(k)} - i\lambda(k)\right],$$

where

$$\lambda^2(k) = \frac{1}{\tau^2}\,\frac{c^2}{4(\omega_0^2 + k^2c^2)c_e^2}.$$

This formula is valid when $\lambda \ll c/c_e \gg 1$. When $\lambda(0) \ll 1$, we see that the damping is small at all frequencies. If $\lambda(0) > 1$, then $\omega_2(k)$ is pure imaginary at low frequencies.

At sufficiently high frequencies ($\omega_2\tau > 1$), however, we have

$$\omega_2 \cong \pm c_e k - i\frac{1}{\tau}$$

and the damping may turn out to be small if $c_e \gg n^{-1/3}/\tau$.

In conclusion, we note that a sufficiently ideal electronic system ($n^{-1/3}e^2 \gtrsim T$), even if it does not form a crystal, can apparently have a shear modulus at sufficiently large frequencies ω, such that the short-range-order realignment does not have time to occur within one period of the oscillations. Then passage of transverse electromagnetic waves with $\omega < \omega_0$ should be observed at these frequencies (as in an "electronic crystal").

[1] D. Pines, Elementary Excitations in Solids, Benjamin, 1963, Ch. III (Russ. Transl. Mir, 1965).
[2] R. A. Coldwell-Horsfall and A. A. Maradudin, J. Math. Phys. 1, 395 (1960).
[3] W. J. Carr, Phys. Rev. 122, 1437 (1961).

CRITICAL CURRENT OF A SUPERCONDUCTING FILM IN A MIXED STATE

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Submitted 11 March 1969
ZhETF Pis. Red. 9, No. 8, 494-496

All papers dealing with the critical current of rigid superconductors contain the statement that the mixed state ($H_{c1} < H_0 < H_{c2}$) is absolutely unstable with respect to a transport current directed perpendicular to the external magnetic field H_0. In other words, this means that the transport current interacting with the superconducting vortices exerts on them Lorentz force and causes them to move in a direction perpendicular to the field and to the current. This gives rise to energy dissipation and destruction of the superconducting state. If the material is inhomogeneous, then the vortices become pinned to the inhomogeneities and nondissipative superconducting flow of the transport current is possible.

We note first of all that the statement that the mixed state is absolutely unstable is, strictly speaking, incorrect. Indeed, it would be valid for an infinite sample, but even the surface of a real sample can serve as the homogeneity on which the vortex motion can become pinned.

We calculate in this paper the critical current for a film placed parallel to an external magnetic field. The film thickness d is assumed small compared with the penetration depth δ_0: $d \ll \delta_0$, but $d \gg \xi(T)$, where $\xi(T) = \delta_0/\kappa$ and $\kappa \gg 1$ (κ is the constant of the Ginzburg-Landau theory [1]). We consider a case when the external magnetic field is $H_0 > H_{c1}(d)$, but $H_0 - H_{c1}(d) \ll H_{c1}(d)$. Here $H_{c1}(d)$ is the first critical field of the film, which was calculated by Abrikosov in [2]. Let the film be parallel to the (yz) plane and bounded by the planes $x = \pm d/2$. The external magnetic field is directed along the oz axis, and the transport current flows in the oy direction.

We consider first the case when there is no transport current. Let us find the free energy of such a configuration of vortices: the axes of all the vortices are parallel to yz, the points of intersection of the axes of all the vortices with the (xy) plane lie on a single line parallel to the oy axis and located a distance x_0 away from it, and the distance a between the axes of the vortices is large compared with δ_0. The solution of the equation for the field

$$\Delta H - H = -\frac{2\pi}{\kappa}\sum_{m=-\infty}^{\infty}\delta(x - x_0)\,\delta(y - ma)$$