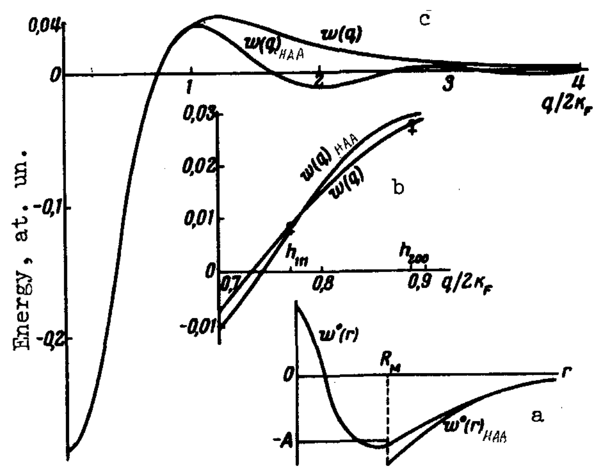

was assumed to be the same as in [4].

For the metals considered, $a > 1$; as seen from (1), this means that in the region $r < r_c$ the electrons are not attracted, but repelled. Figure 1a shows schematically for comparison, plots of $w^o(r)$ and $w^o(r)_{HAA}$. Figures 1b and 1c show the form factor $w(q)$ of our potential and $w(q)_{HAA}$ for aluminum. The points and crosses denote the experimental values taken respectively from [9] and [8].

An important characteristic of the pseudopotential is the quantity q_0 - the zero of $w(q)$ (the first zero in the case of the HAA form factor) [7]. As seen from the table, the values of q_0 corresponding to this potential are very close to $q_{0\ HAA}$ (the latter are regarded as reliable and are frequently used to "trim" other model potentials). We see also that the values of $w(q)$ at those points not used in (5) are in good agreement with the corresponding experimental values.

It is hoped that the proposed model pseudopotential proves useful in the study of many properties of metals, including atomic properties.

The authors are grateful to G. P. Motulevich and L. V. Keldysh for a discussion.

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CORRECTIONS TO THE GELL-MANN - SHARP - WAGNER MODEL FOR THE MESON DECAYS $\omega \to 3\pi$, $\omega \to \pi\gamma$, $\pi \to \gamma\gamma$

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Recent experiments on vector-meson production in colliding electron-positron beams, and also the accumulation of a large amount of data on vector-meson photoproduction, have led to an increased interest in a verifications of the vector-dominance model, which is in satisfactory agreement with the majority of the experimental results [1, 2]. For the ratios of the probabilities of the decays $\omega \to 3\pi$, $\omega \to \pi\gamma$, and $\pi^0 \to 2\gamma$, the predictions obtained in the vector-dominance model[1)] contradict the experimental data [2, 4]. In this work we consider the corrections in the GSW model, resulting from the departure of the virtual particles from the mass shell. To this end, we introduce the form factor of the $\omega\rho\pi$ vertex, viz., $F(p_\omega^2/m_\omega^2,\ p_\rho^2/m_\rho^2)$, $F(1, 1) \equiv 1$. We introduce no form factors for the transitions $\rho \to \gamma$ and $\omega \to \gamma$, since an analysis of the data on processes in which the form factor of the transition $\rho \to \gamma$ might play any role shows that, within the limits of experimental errors, this form factors changes little in the interval $0 \leq p_\rho^2 \leq m_\rho^2$ [1]. We propose that the form factor of the $\omega \to \gamma$ behaves similarly. The role of the ρ-meson form factor can be estimated by using the Breit-Wigner approximation, since the ρ meson does not go too far off the mass shell in the $\omega \to \rho\pi \to 3\pi$ decay. The corresponding corrections are small, and will henceforth be neglected. Under the foregoing assumptions, we first estimate phenomenologically the dependence of $F(x,y)$ on $x \equiv p_\omega^2/m_\omega^2$ and $y \equiv p_\rho^2/m_\rho^2$, using the so-refined GSW model and the experimental data, and then consider a simple dynamic model in which F can be calculated when $x = y$.

In the notation of [3], we obtain

$$\Gamma(\omega \to 3\pi) = \frac{m_\omega}{3}\,\frac{\gamma^2_{\rho\pi\pi}}{4\pi}\,\frac{f^2_{\omega\rho\pi}}{4\pi}\,m_\pi^2\,\frac{(m_\omega - 3m_\pi)^4}{(m_\rho^2 - 4m_\pi^2)^2}\,\frac{3{,}56}{\sqrt{3}}\,\overline{F^2(1, y)}, \tag{1}$$

$$\Gamma(\omega \to \pi\gamma) = \frac{m_\omega}{3}\,a\,\frac{f^2_{\omega\rho\pi}}{4\pi}\,m_\pi^2\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\frac{(m_\omega^2 - m_\pi^2)^3}{32 m_\omega^4 m_\pi^2}\,F^2(1,0), \tag{2}$$

$$\Gamma(\pi^0 \to 2\gamma) = m_\pi\,\frac{a^2}{64}\,\frac{f^2_{\omega\rho\pi}}{4\pi}\,m_\pi^2\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\left(\frac{\gamma_\omega^2}{4\pi}\right)^{-1}F^2(0,0). \tag{3}$$

Expression (1) contains a certain mean value of the square of the form factor $F(1, y)$, which we have designated $\overline{F^2(1, y)}$. To estimate the change of $F(x, y)$, we assume a very simple linear relation that is symmetrical in x and y:

$$F(x, y) = F(0,0)\,[1 + \lambda(x + y)]\ ,\quad 0 \leqslant x \leqslant 1,\quad 0 \leqslant y \leqslant 1. \tag{4}$$

To estimate λ, we use two sets of experimental data:

1) $\gamma_\rho^2/4\pi = 0{,}53 \pm 0{,}04$; $\gamma_\omega^2/4\pi = 4{,}69\,^{+1.24}_{-0.81}$; $\frac{\gamma^2_{\rho\pi\pi}}{4\pi} = \frac{1}{4}(2{,}12 \pm 0{,}16)$ [1]

2) $\gamma_\rho^2/4\pi = 0{,}51 \pm 0{,}03$; $\gamma_\omega^2/4\pi = 3{,}7 \pm 0{,}7$; $\frac{\gamma^2_{\rho\pi\pi}}{4\pi} = \frac{1}{4}(2{,}10 \pm 0{,}11)$ [2].

The value of γ_ρ agrees with the vector-dominance model for the photoproduction and for the form factor of the pion. (We note, however, that from the global mean value [4] for $\Gamma(\rho \to \pi\pi)$ we obtain $\gamma^2_{\rho\pi\pi}/4\pi = 1/4(2.42 \pm 0.40)$, which yields somewhat different numerical values for the quantities considered below).

Comparing now the ratios $\Gamma(\omega \to \pi\gamma)/\Gamma(\pi^0 \to 2\gamma)$ and $\Gamma(\omega \to 3\pi)/\Gamma(\pi^0 \to 2\gamma)$ obtained from (1) - (3) with the experimentally obtained ratios, we obtain for the data set 1:

[1)] See [3]. We shall henceforth call this model the GSW model.

$$\frac{F^2(1,0)}{F^2(0,0)} = 2,05 \pm 0,60, \quad (5a); \qquad \frac{\overline{F^2(1,y)}}{F^2(0,0)} = 3,08 \pm 0,90, \tag{5b}$$

and for the data set 2:

$$\frac{F^2(1,0)}{F^2(0,0)} = 2,60 \pm 0,60, \quad (6a); \qquad \frac{\overline{F^2(1,y)}}{F^2(0,0)} = 4,23 \pm 1,00, \tag{6b}$$

where we have used the experimental values [1, 2, 4] $\Gamma(\omega \to 3\pi) = 11.0 \pm 1.2$ MeV, $\Gamma(\omega \to \pi\gamma) = 1.13 \pm 0.15$ MeV, and $\Gamma(\pi^0 \to 2\gamma) = 7.4 \pm 1.5$ eV.

From (5a) we obtain $\lambda \equiv \lambda_a^{(1)} = 0.43 \pm 0.19$, and from (6a) it follows that $\lambda \equiv \lambda_a^{(2)} = 0.61 \pm 0.20$. To be able to use relations (5b) and (6b), it is necessary to find $\overline{F^2(1, y)}$. It can be shown that

$$\overline{F^2(1,y)} = F^2(0,0)(1+2\lambda)^2 \left[1 - \frac{4\lambda}{1+2\lambda}\epsilon\right], \tag{7}$$

where ε is approximately equal to 0.1. Thus, $\overline{F^2(1, y)} \simeq F^2(1, 1)$, confirming, in particular, the assumption $p_\rho^2 \sim m_\rho^2$ made above. Substituting (7) in (5) and (6) we get $\lambda_b^{(1)} = 0.41 \pm 0.12$ and $\lambda_b^{(2)} = (0.58 \pm 0.12)$. The obtained values of $\lambda_{a,\,b}^{(1,2)}$ agree within the limits of experimental errors. Since one parameter λ has been determined from the two independent ratios $\Gamma(\omega \to \pi\gamma)/\Gamma(\pi^0 \to 2\gamma)$ and $\Gamma(\omega \to 3\pi)/\Gamma(\pi^0 \to 2\gamma)$, we can conclude that the phenomenological model proposed by us "works" unexpectedly well and yields $\lambda \sim 0.5$.

The obtained dependence of $F(x, y)$ on x and y may appear also in other processes. Great interest attaches from this point of view to the decay $\pi^0 \to e^+e^-$, the probability of which can be estimated in the model illustrated in Fig. 1a. When $F(x, y) \simeq F(0, 0)$, calculation of this diagram yields [6] $\Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to 2\gamma) = (5 - 6) \times 10^{-8}$. The use of the form factor $F(x, y)$ from (4) can greatly increase this ratio (by approximately 3 - 5 times). Similarly, allowance for $F(x, y)$ decreases the prediction of the simple vector-dominance model for the $\rho \to 4\pi$ decay (Fig. 1b). These effects are worthy of a separate study and will be discussed in another paper.

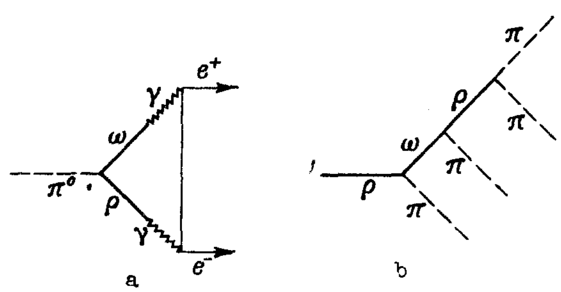

Fig. 1

Let us consider, in conclusion, a simple dynamic model in which the dependence of $F(x, y)$ on x and y can be obtained. The $\omega\rho\pi$ interaction is described by the Lagrangian $L = f_{\omega\rho\pi}\varepsilon^{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu\partial_\lambda\rho_\sigma\pi$. The simplest way of finding $F(x, y)$ reduces to a calculation of a diagram of third order in $f_{\omega\rho\pi}$. However, since this interaction is non-renormalizable, the result depends on the cutoff. In addition, the $f_{\omega\rho\pi}$ coupling constant is not small and it is therefore necessary to take into account also the contributions of higher-order diagrams. Starting from these considerations, we have estimated $F(x, y)$, calculating the sum of an inifnite set of diagrams of the "ladder" type (Fig. 2).

We put $$\Gamma^{\mu\nu}_{\omega\rho\pi}(p_\omega, p_\rho) = f_{\omega\rho\pi}\epsilon^{\mu\nu\lambda\sigma}p_{\omega,\lambda}p_{\rho,\sigma}\kappa \times F\left(\frac{p_\omega^2}{m_\omega^2}, \frac{p_\rho^2}{m_\rho^2}\right).$$

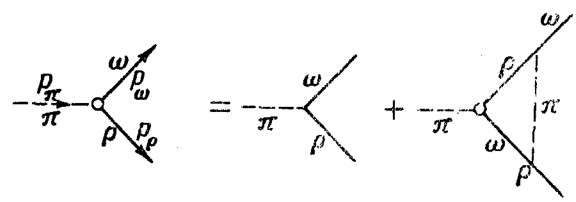

Fig. 2

F(x, y) satisfies a linear integral equation ($\Gamma_{\omega\rho\pi}$ is represented by a circle in Fig. 2). To simplify the problem, we use the approximations $p_\pi = p_\omega + p_\rho \approx 0$ and $m_\pi \approx 0$, which correspond to the soft-pion approximation. The equation for $F(x) \equiv F(x, x)$ then coincides with the equation for the vertex function, investigated in [5], and can be solved by the same methods. If we use the modified perturbation theory [5], then we can fined F in the form of an expansion in posers of $f \equiv (1/12)(f^2_{\omega\rho\pi}/4\pi)m_\pi^2 \approx 0.50$ and log f. Taking only terms of order f and f^2 in F, we obtain $F(1.1)/F(0,0) \approx 1.23$. A second-order approximation gives a good result when $f \leq 0.35$. A rough estimates of the higher-order contributions at f = 50 yields F(1, 0)/F(0,0) 1.3. Thus, by using the linear extrapolation (4) we obtain $\lambda \sim 0.15$. (A detailed description of the calculation will be presented in a separate article.) Thus, the considered simple model agrees qualitatively with experiment. For a quantitative comparison of theory with experiment it is necessary to take into account diagrams not of the ladder type, and also to determine the corrections necessitated when $p_\pi \neq 0$. We note also that the value of λ depends strongly on $\Gamma(\pi^0 \to 2\gamma)$. In particular, if we accept the recently obtained [1,7] new value $\tau_{\pi^0} = (0.6^{+0.2}_{-0.08}) \times 10^{-16}$ sec and $\Gamma(\pi^0 \to 2\gamma) = (11^{+1.6}_{-2.8})$ eV, then we get $\lambda_a^{(1)} = (0.18 \pm 0.18)$, $\lambda_b^{(1)} = (0.23 \pm 0.11)$, $\lambda_a^{(2)} = (0.32 \pm 0.17)$, and $\lambda_b^{(2)} = (0.37 \pm 0.11)$, which is quite close to the prediction of our model.

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ERRATA

In the article by E. M. Barkhudarov et al., Vol. 9, No. 5, the formula on p. 163 should read "$f_0 = 1/2L\sqrt{kT_e/m_i}$," and not "$f_0 = 1/L\sqrt{kT_e/m_i}$."

The authors of the article by I. N. Erofeeva et al., Vol. 9, No. 7, p. 232, are affiliated with the Moscow State University and not with the P. N. Lebedev Physics Institute.

Figure 2 of the article by I. R. Gekker and O. V. Sizukhin, Vol. 9, No. 7, p. 245, is upside down. Its correct form is shown on the next page.