[1] Yu. M. Aivazyan, V. A. Belyakov, Zh. Eksp. Teor. Fiz. 56 346 (1969) [Sov. Phys.-JETP 29, (1969)].
[2] G. V. Smirnov, V. V. Sklyarevskii, R. A. Voskanyan, and A. N. Artem'ev, ZhETF Pis. Red. 9, 123 (1969) [JETP Lett. 9, 70 (1969)].
[3] T. Mizogchi and M. Tanaka, J. Phys. Soc. Japan 18, 1301 (1963)

NONLINEAR EFFECTS IN A SUPERCONDUCTOR IN AN ALTERNATING FIELD

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When the temperature of a superconductor approaches the critical value, nonlinear effects assume a role even in weak alternating external field, since the critical field of the superconductor tends to zero. Using the calculation method developed in [1], we investigated several such nonlinear effects, viz., generation at the frequency $3\omega_0$ and generation of combination frequencies, and we calculated the surface impedance of a superconductor in a strong alternating electromagnetic field.

It was assumed that the superconductor is pure with respect to its equilibrium properties, $\ell \gg \xi_0$, but contains a small amount of impurities, so that the skin effect is normal, $\delta_s \gg \vec{\ell}$ (ℓ is the mean free path, δ_s the depth of penetration of the field, and ξ_0 the correlation parameter). In the temperature region $1 - T/Tc \ll \kappa^2$ this makes it possible to write down for the current the expression (τ is the free path time):

$$j(q\omega) = -\frac{c}{4\pi} K(\omega) A(q\omega) = -\frac{Ne^2}{mc}\left[\frac{7\xi(3)}{4}\left(\frac{\Delta}{\pi T}\right)^2 - i\omega\tau\right] A(q\omega). \quad (1)$$

In order for the question to be of experimental interest, the region $1 - T/T_c \ll \kappa^2$ is assumed to be sufficiently large ($\kappa \sim 1$). It is known [2] that the behavior of a superconductor in an alternating field depends on the ratio of the frequency of the external field ω_0 to the frequencies Ω_1 and Ω_0 characterizing the dynamic behavior of the electromagnetic field and of the parameter $\Delta(r, t)$. In our case, using (1) and the formulas of [3], it can be shown that

$$\Omega_1 \sim \frac{1}{\tau T_c}\frac{\Delta^2}{T_c}, \quad \Omega_0 \sim \frac{\Delta^2}{T_c}, \quad \Omega_1 \ll \Omega_0. \quad (2)$$

Using the results of [1], we can also show that at external-field frequencies $\omega_0 \ll \Omega_0$ the gap responds to the instantaneous value of the field, and satisfies at temperatures $1 - T/T_c \ll \kappa^2$ the Ginzburg-Landau equation, in which, however, the field depends on the time as a parameter. At frequencies $\omega_0 \gg \Omega_0$ we can use for the alternating part of $\Delta(r, t)$ the equation obtained in [4]. These equations form, in conjunction with Maxwell's equation, a closed nonlinear system whose solution was obtained by perturbation theory using the small parameter H_0/H_c, where H_0 is the amplitude of the external field and $H_c(T)$ is the critical field.

1. Third-harmonic generation. If an electromagnetic wave of frequency ω_0 is incident on the superconductor, then the presence of the harmonic $2\omega_0$ in the oscillations of the gap gives rise to a current of frequency $3\omega_0$, leading to radiation of the corresponding field harmonic. We calculated the power conversion coefficient $\eta(3\omega_0) = |\alpha(3\omega)|^2 = |E_{ref}(3\omega_0)/E_{inc}(\omega_0)|^2$ for the case of a half-space. The expression obtained for $\alpha(3\omega_0)$ is:

$$\alpha(3\omega_o) = i\frac{3\omega_o}{c}\left(\frac{H_o}{H_c}\right)^2 \frac{\kappa^2}{\delta_Л^4}\frac{\gamma(3\omega_o)+\gamma(\omega_o)}{\gamma(3\omega_o)\gamma^2(\omega_o)}$$

$$\int_{-\infty}^{+\infty}\frac{dq/2\pi}{[q^2+\beta^2][q^2+(\gamma(3\omega_o)+\gamma(\omega_o))^2][q^2+(2\gamma(\omega_o))^2]} \tag{3}$$

Here

$$\gamma = \sqrt{K(\omega)} \quad \mathrm{Re}\gamma > 0 \quad \delta_Л = \delta_o(1 - T/T_c)^{-1/2} = 1{,}04\kappa\xi_o(1-T/T_c)^{-1/2}$$

$$\beta = \begin{cases} \dfrac{\sqrt{2}\kappa}{\delta_Л} & \omega_o << \Omega_o \\ \sqrt{\dfrac{\omega_o r_o}{2}}(1-i) & \Omega_o << \omega_o \end{cases} \qquad r_o = \frac{\pi\kappa^2}{4\delta_o^2 T_c}.$$

The integral in (3) can be calculated in elementary fashion, but we shall not present the result, which is too unwieldy. We confine ourselves to the limiting cases (H_{co} is the critical field at T = 0)

$$1)\ \omega_o << \Omega_1, \quad \gamma(3\omega_o) = \gamma(\omega_o) = \delta_Л^{-1}:$$

$$\eta_1 = 9{,}8\cdot 10^{-4}\left(\frac{\omega_o}{c}\delta_o\right)^2\left[\kappa\frac{\kappa+2\sqrt{2}}{(\kappa+\sqrt{2})^2}\right]^2\left(\frac{H_o}{H_{co}}\right)^4\left(1-\frac{T}{T_c}\right)^{-5}. \tag{4}$$

The quantity η_1 is related in obvious fashion to the ordinary formulas for the correction to the depth of penetration of an optical field [5].

$$2)\ \Omega_1 << \omega_o << \Omega_o, \quad \gamma(\omega_o) = (1-i)\delta_s^{-1} = (1-i)\frac{1}{c}\sqrt{2\pi\omega_o\sigma}:$$

$$\eta_2 = 5{,}5\cdot 10^{-6}\left(\frac{\omega_o}{c}\delta_o\right)^2\kappa^2\left(\frac{H_o}{H_{co}}\right)^4\frac{\delta_s}{\delta_o}^{12}(1-T/T_c). \tag{5}$$

$$3)\ \Omega_o << \omega_o:$$

$$\eta_3 = 1{,}1\cdot 10^{-5}\frac{\omega_o}{c^2 r_o}\kappa^4\left(\frac{H_o}{H_{co}}\right)^4\left(\frac{\delta_s}{\delta_o}\right)^{12}\left(1-\frac{T}{T_c}\right)^2. \tag{6}$$

It is seen from formulas (4) - (6) that η, as a function of the temperature, has a sharp maximum provided $\omega_{0\,max} \sim \Delta^2/\tau T_c^2 = (1/\tau)8\pi^2/7\xi(3)\ (1 - T/T_c)$. At these frequencies $\omega_0 >> \omega_{0\,max}$ the field penetrates into the metal to a depth $\delta_s << \delta_\ell$. Inasmuch as Ω_1 and Ω_0 depend on the temperature, we see that by choosing a sufficiently low frequency of the external field (at $1 - T/T_c \sim 10^{-2}$ and $\ell = v\tau \sim 10^{-3}$ cm we have $\omega_{0\,max} \sim 10^8 - 10^9$ Hz), it is possible to realize any of the aforementioned three situations by varying the temperature.

By using the "joining" method [6], it is possible to calculate η for a large field amplitude in the frequency range $\Omega_1 << \omega_0 << \Omega_0$. In this frequency range, the field is determined in first approximation by the skin effect, and the superconductivity is not destroyed even in strong fields. The spectrum of the reflected signal contains all the frequencies $\omega_0(2k + 1)$ $(k = 0, \pm 1, \pm 2, \ldots)$. If $\kappa(H_0 H_c)^2(\delta_s/\delta_\ell)^3 \gg 1$, we have

$$\eta = 2{,}9\cdot 10^4\left(\frac{\omega_o}{c}\delta_o\right)^2 \frac{1}{\kappa^4}\left(\frac{H_{co}}{H_o}\right)^8\left(\frac{\delta_o}{\delta_s}\right)^6 (1 - T/T_c)^4 B(\kappa),$$

$$B(1) = 1, \qquad B(\kappa) \to \frac{1{,}5\kappa^2}{(\sqrt{2} + 1)^{2k}} \quad \text{as} \quad \kappa \to \infty. \tag{7}$$

The conversion of the energy of the incident wave into the energy of the higher harmonics leads to an appreciable change of the character of the reflection at the fundamental frequency ω_0. We present the result for the surface impedance in a strong field:

$$\xi = (1 - i)\frac{\omega_o \delta_s}{2c}\left[1 + i\,\frac{65}{\kappa^2}\left(\frac{H_{co}}{H_o}\right)^4\left(\frac{\delta_o}{\delta_s}\right)^4\left(1 - \frac{T}{T_c}\right)^2\right] \tag{8}$$

2. Combination frequencies. When two electromagnetic waves with close frequencies ω_1 and ω_2 are incident on a superconductor, reflection is produced at combination frequencies, particularly at the frequency $2\omega_1 - \omega_2$. In analogy with the preceding case, we calculated the conversion coefficient $\eta(2\omega_1 - \omega_2)$, which also has a sharp temperature maximum under the condition $\omega_1\omega_2 \sim \Omega_1$. We present the results for the most interesting case, when $\omega_1 - \omega_2 \ll \Omega_1$

1) $\omega_1, \omega_2 \gg \Omega_1$:

$$\eta_1 = 4{,}4\cdot 10^{-4}\left(\frac{\omega_1}{c}\delta_o\right)^2\left[\kappa\frac{\kappa + 2\sqrt{2}}{(\kappa + \sqrt{2})^2}\right]^2\left(\frac{H_1 H_2}{H_{co}}\right)^4 (1 - T/T_c)^5; \tag{9}$$

2) $\omega_1, \omega_2 \gg \Omega_1$:

$$\eta_2 = 1{,}4\cdot 10^{-5}\left(\frac{\omega_1}{c}\delta_o\right)^2 \kappa^2\left(\frac{H_1 H_2}{H_{co}}\right)^4\left(\frac{\delta_s}{\delta_o}\right)^{12} (1 - T/T_c). \tag{10}$$

In conclusion we note that experiments in which similar phenomena were observed were performed, unfortunately, only on films [7].

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[1] L. P. Gor'kov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. 54, 612 (1968) [Sov. Phys.-JETP 27, 328 (1968)].
[2] L. P. Gor'kov and G. M. Eliashberg, ZhETF Pis. Red. 8, 329 (1968) [JETP Lett. 8, 202 (1968)].
[3] M. P. Kemoklidze, Zh. Eksp. Teor. Fiz. 53, 1362 (1967) [Sov. Phys.-JETP 26, 792 (1968)].
[4] E. Abrahams and T. Tsuneto, Phys. Rev. 152 , 416 (1966).
[5] V. L. Ginzburg and and L. D. Landau, Zh. Eksp. Teor. Phys. 20, 1064 (1950).
[6] L. P. Gor'kov and G. M. Eliashberg, ibid. 55, 2430 (1968) [28,(1969)].
[7] K. Rose and M. D. Sherrill, Phys. Rev. 145, 179 (1966).