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SELF-FOCUSING OF ULTRASHORT LASER PULSES

A. A. Abramov, V. N. Lugovoi, and A. M. Prokhorov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
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The self-focusing phenomenon has heretofore been investigated theoretically only for stationary [1-3] or nonstationary [4] boundary conditions. Such a formulation of the problem corresponds to the case when the duration of the laser pulse is sufficiently large, viz., the length of the light train determined by this duration should be much larger than the width of the nonlinear-medium layer through which the laser beam passes and in which its behavior is of interest to us. We consider in this paper the propagation, in a nonlinear medium, of "ultrashort" light pulses, with train lengths much shorter than (or of the order of) the width of the layer in which the self-focusing takes place. It is assumed here that the nonlinearity of the medium is due to the Kerr effect:

$$\epsilon = \epsilon_o + \frac{1}{2}\epsilon_2 |E|^2 \tag{1}$$

(ε is the dielectric constant, $|E|$ the amplitude of the electric-field oscillations). The electric field $\mathcal{E}(\vec{r}, t)$ is assumed scalar, for simplicity, i.e., satisfying the condition

$$\Delta \mathcal{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\epsilon \mathcal{E}) = 0. \tag{2}$$

Assuming

$$\mathcal{E} = \frac{1}{2} E(r, t)\, e^{i(kz - \omega t)} + \text{c.c.}, \quad \left(k = \frac{\omega}{c}\sqrt{\epsilon_o}\right) \tag{3}$$

we find an equation for the new unknown function E:

$$\Delta E + 2ik\frac{\partial E}{\partial z} + \left(\frac{\omega}{c}\right)^2 \epsilon' E + \frac{\epsilon_o + \epsilon'}{c^2}\left(2i\omega\frac{\partial E}{\partial t} - \frac{\partial^2 E}{\partial t^2}\right) + \frac{2}{c^2}\frac{\partial \epsilon'}{\partial t}\left(i\omega E - \frac{\partial E}{\partial t}\right) - \frac{1}{c^2}\frac{\partial^2 \epsilon'}{\partial t^2} E = 0, \tag{4}$$

where $\varepsilon' = (1/2)\, \varepsilon_2\, |E|^2$. We shall assume that the conditions

$$\Delta t_x \gg T, \quad \Delta z_x \gg \lambda, \quad \epsilon' \ll \epsilon_o, \tag{5}$$

are satisfied for all $\vec{r}$ and t; here Δt_x and Δz_x are the characteristic time and the characteristic length of variation of the field E as functions of t and z, respectively; $T = 2\pi/\omega$ is the period of the optical oscillation; $\lambda = 2\pi/k$ is the length of the light wave. Under conditions (5), some of the terms of (4) are negligibly small. Omitting these terms, we arrive at the following equation for E:

$$\Delta_\perp E + 2ik\left(\frac{\partial E}{\partial z} + \frac{1}{v}\frac{\partial E}{\partial t}\right) + n_2 k^2 |E|^2 E = 0, \tag{6}$$

where
$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad v = \frac{c}{n_o}, \quad n_o = \sqrt{\epsilon_o}, \quad n_2 = \frac{\epsilon_2}{2\epsilon_o}.$$

We are interested in a solution E(x, y, z, t) of this equation (at $z > 0$) satisfying the boundary condition

$$E(x, y, 0, t) = \phi(x, y, t), \tag{7}$$

where ϕ is a specified function of the transverse coordinates and the time, determined respectively by the transverse distribution and by the time dependence of the laser pulse incident (from the half-space $z < 0$) on the boundary $z = 0$.

It is easy to verify that the change of variable $t = \tau + z/v$ in (5) yields the following equation for the function E:

$$\Delta_\perp E + 2ik\frac{\partial E}{\partial z} + n_2 k^2 |E|^2 E = 0 \tag{8}$$

with boundary condition

$$E|_{z=0} = \phi(x, y, \tau). \tag{9}$$

This problem is stationary, and the variable τ plays only the role of a parameter in the boundary condition. Therefore, if a family of solutions of the stationary self-focusing problem (8) - (9) is known (corresponding to all possible values of τ), then by putting $\tau = t - z/v$ in this family we obtain the solution of the problem (6) - (7) of interest to us. The family of solutions of Eq. (8), corresponding to the stationary self-focusing of an axially symmetrical beam with a Gaussian initial intensity distribution in the transverse cross section and with a plane phase front, was obtained in [2][1].

On this basis we obtain immediately the following general conclusion: The self-focusing of a light beam (whose maximum power exceeds the usual critical power) occurs even when the length of the corresponding light train is much shorter than the characteristic self-focusing length (determined formally from the maximum power). The resultant general picture of the phenomenon is as follows: Just as for stationary (quasistationary) self-focusing, the main feature of this picture is the existence of a series of focal points (i.e., region of very small dimensions and high energy concentrations) on the beam axis. For ultrashort laser pulses, these points occur at definite instants of time and may multiply as they occur, moving along the z axis both in the beam propagation direction and in the opposite direction.

We present a method for determining the coordinates of all the focal points (along the z axis) at any instant of time t_0, if the incident pulse is specified by the boundary condition $E|_{z=0} = \psi(t)\exp(-r^2/2a)^2$. To this end, we plot, in the coordinates

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[1] A similar family of solutions, corresponding to a spherical initial front, will be presented in [5].

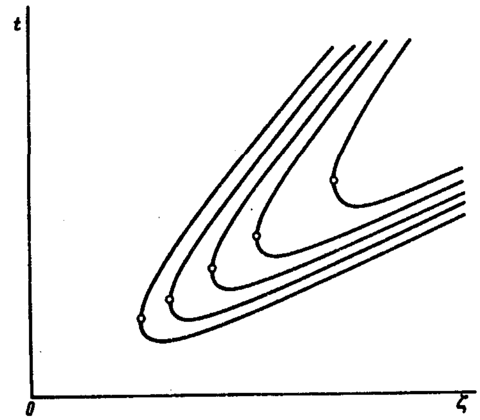

Fig. 1

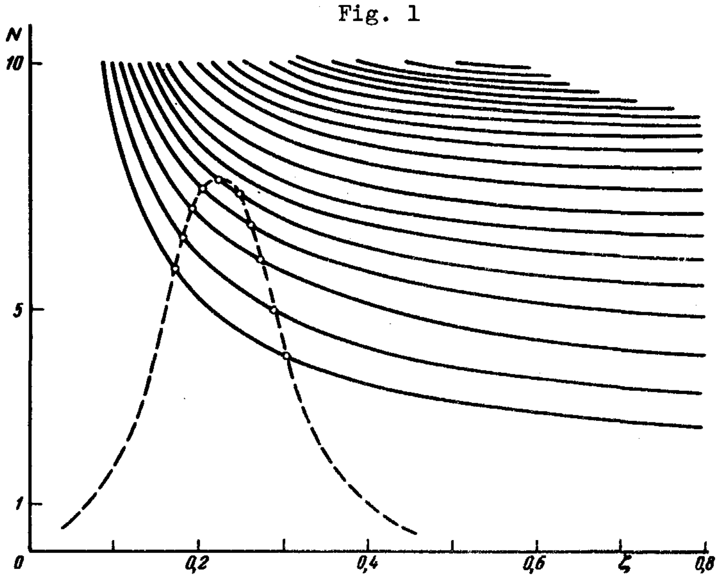

Fig. 2

$$\zeta = \frac{z}{ka^2}, \quad N = \frac{E_0}{E_{cr}} \left(E_{cr} = \frac{1}{\sqrt{n_2 (ka)^2}}\right)$$

the aggregate of the $N = N\,N_m(\zeta)$ curves (solid curves of Fig. 1) determined by the plot given in [2] (which represents the "inverse" dependence of Nz_m/ka^2 on N). We plot in the same coordinates the function

$$N = \frac{1}{E_{cr}} \left| \psi\left(t_0 - \frac{ka^2}{v}\zeta\right)\right|$$

(see the dashed curve of Fig. 2). The values of ζ corresponding to the points of intersection of the solid and dashed curves determine the location (in a suitable scale) of the focal points on the beam axis at the instant t_0. If we recognize that when t_0 changes the dashed curve shifts along the ζ axis (with velocity v/ka^2), remaining similar to itself, then it is easy to explain the appearance of the focal points and the time dependence of their positions. This picture is shown in Fig. 2. We see that, just as in quasistationary self-focusing, there are always "turning points" at which the focal points are stopped (these are denoted by circles in Fig. 2).

It is also easy to see that in self-focusing of pulses of different durations, the speeds of the focal points can be either larger or smaller than the speed of light c. In the self-focusing of very short pulses, the longitudinal dimensions of the focal points will be much shorter than the analogous dimensions in stationary self-focusing. Formally (i.e., by virtue of (8) - (9)), the indicated dimensions may turn out to be also so small that the characteristic time of the change of $|E|^2$ at a fixed point on the z axis (when the focal point passes rapidly through it) is smaller than the characteristic time of establishment of the Kerr effect in the medium. It is clear that in this case the minimal dimension of the focal point (and accordingly the actually attainable concentration of energy in it) will be limited by the time of establishment of the Kerr effect, which in turn can be used in practice for an estimate of this time. In general, the presence of additional phenomena at the focal points (such as breakdown, stimulated Raman and Mandel'shtam-Brillouin scattering, etc) may influence a number of quantitative characteristics of the phenomena under consideration, but introduces no qualitative changes in the obtained picture.

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NONLINEAR RADIAL SELF-FOCUSING OF A MODULATED ELECTRON BEAM IN A PLASMA

V. B. Krasovitskii

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The interaction of a plasma with a strongly modulated electron beam, constituting a sequence of charged discs with a surface charge density $\sigma(r)$ located at distances ℓ from one another and moving with velocity v_0, can be described by introducing into the Maxwell equations for the fields produced by the beam the dielectric constant of the plasma

$$\epsilon(\omega_M) = 1 - \frac{\omega_p^2}{\omega_M^2}.$$

This quantity describes sufficiently well the electromagnetic properties of the plasma if the beam modulation frequency $\omega_M = 2\pi v_0/\ell$ is close to the plasma frequency ω_p: $|\omega_M - \omega_p| \ll \omega_p$. Since the frequency ω_M is determined only by the beam parameters and does not depend on the plasma density, the dielectric constant of the plasma is negative when the condition $\omega_M < \omega_p$ is satisfied, and the Coulomb repulsion of the beam electrons gives way to attraction. When the transverse dimensions r_0 of the plasmoids are large compared with the distances between them, $r_0 \gg \ell$, and consequently the field produced by the plasmoids