$\mathcal{E} < 0.1$ V/cm. Experimental studies of the shift of the energy levels of atomic positronium in gases (so far there are experimental data only for argon at pressures somewhat higher than atmospheric and at a temperature 300°K [11]) can answer the question concerning the concrete substances in which the indicated value of $\mathcal{E}$ is realized.

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STRUCTURE OF LAMB DIP FOR LONG-LIVED SYSTEMS IN SPATIALLY BOUNDED FIELDS

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The most promising method of stabilizing the frequency of laser radiation is based on the use of an absorbing gas cell [1 - 3]. The spectral width of the output-power peak, against which the stabilization is effected, is determined by the time of coherent interaction of the atomic system with the field. In this connection, particular interest attaches to the long-lived systems, i.e., with small level width Γ. Molecule-beam lasers have been analyzed in a number of papers (cf., e.g., [4, 5]). The spectral width of the power peak is determined here only by the time of flight $\bar{\tau}$ of the light wave for the average thermal velocity $\bar{v}$, if $\Gamma \ll 1/\tau$. This raises naturally the question whether this result is general or whether it is possible to construct lasers in which the parameters of the spectral structures are determined by the value of Γ in spite of the fact that $\Gamma \ll 1/\tau$. It will be shown below that in ordinary (not beam) systems, at a certain field configuration, the decisive role is played not by the mean-thermal atoms but by the slow ones, for which the time of flight is larger than or of the order of $1/\Gamma$. Consequently, such lasers can be constructed.

The stationary equations for the density matrix, in the case of a standing monochromatic wave, are given by

$$\left(\Gamma_j + v\frac{\partial}{\partial z} + u\frac{\partial}{\partial x}\right)\rho_j = q_j(u, v) \pm 2\,\mathrm{Re}\{i\,G(x)\,\rho_{mn}\cos kz\};\quad j = m, n$$

$$\left(\Gamma - i\,\Omega + v\frac{\partial}{\partial z} + u\frac{\partial}{\partial x}\right)\rho_{mn} = iG(x)\,(\rho_m - \rho_n)\cos kz;\quad \Omega = \omega - \omega_{mn}, \tag{1}$$

where v and u are the z and x components of the velocity and q_j is the rate of excitation of the level j. The dependence of the field amplitude on the transverse coordinate x is chosen for simplicity in the form

$$G(x) = \begin{cases} G \text{ for } 0 \leqslant x \leqslant \ell \\ 0 \text{ for other x.} \end{cases} \tag{2}$$

The power of the stimulated emission (with allowance for the first-order correction for saturation), averaged over v and x in the interval $(0, \ell)$, is

$$\langle p(\tau)\rangle_v = p_0\Big\{1 - \frac{G^2}{4}\Big[\frac{1}{\Gamma_m\Gamma}\Big(1 + \frac{\Gamma^2}{\Gamma^2+\Omega^2}\Big) - \frac{\Gamma^2(1-e^{-\Gamma_m\tau}) - \Gamma_m^2(1-e^{-\Gamma\tau})}{\tau\,\Gamma_m^2\,\Gamma^2(\Gamma-\Gamma_m)} -$$

$$- \mathrm{Re}\,\frac{(\Gamma+i\,\Omega)^2(1-e^{-\Gamma_m\tau}) - \Gamma_m^2(1-e^{-(\Gamma+i\Omega)\tau})}{\tau\,\Gamma_m^2(\Gamma+i\,\Omega)^2(\Gamma+i\,\Omega-\Gamma_m)} + \text{analog. terms with } \Gamma_n\Big]\Big\}, \tag{3}$$

$$\tau = \frac{\ell}{u};\quad p_0 = \hbar\omega_{mn}\frac{G^2 N\sqrt{\pi}}{kv}\,e^{-\left(\frac{\Omega}{kv}\right)^2}.$$

Let us analyze Eq.(3) in the case $\Gamma\bar{\tau} \equiv \Gamma\ell/\bar{v} \ll 1$ of interest to us. For atoms with velocities $u > \Gamma\ell$ we have

$$\langle p(\tau)\rangle_v = p_0\left(1 - \frac{G^2}{6}\frac{\ell^2}{u^2}\right). \tag{4}$$

From atoms with $u < \Gamma\ell$ we have

$$\langle p(\tau)\rangle_v = p_0\Big\{1 - \frac{G^2}{4\Gamma}\Big(\frac{1}{\Gamma_m} + \frac{1}{\Gamma_n}\Big)\Big(1 + \frac{\Gamma^2}{\Gamma^2+\Omega^2}\Big)\Big\}. \tag{5}$$

The frequency dependence for slow atoms coincides, as it should, with that of the usual "homogeneous" problem. Let us compare the orders of magnitude of the contributions of the various atoms to that term of (3) which is responsible for the Lamb dip, averaging over the transverse velocities u with various characteristic distributions W(u) ($\Gamma = \Gamma_m = \Gamma_n$; $\Omega = 0$):

$W(u)$	$u < \Gamma\ell$	$u > \Gamma\ell$	
$\frac{1}{\sqrt{\pi}\,v}\,e^{-\frac{u^2}{\bar{v}^2}}$	$G^2\frac{\bar{\tau}}{\Gamma}$	$\frac{1}{3}G^2\frac{\bar{\tau}}{\Gamma}$	(6)
$\frac{2u}{\bar{v}^2}\,e^{-\frac{u^2}{\bar{v}^2}}$	$G^2\bar{\tau}^2$	$\frac{G^2}{4}\ln\frac{1}{\Gamma\bar{\tau}}$	(6')
$\frac{4u^2}{\sqrt{\pi}\,\bar{v}^3}\,e^{-\frac{u^2}{\bar{v}^2}}$	$\frac{1}{2}G^2\Gamma\bar{\tau}^3$	$\frac{1}{3}G^2\bar{\tau}^2$.	(6")

It is seen from (6) that only in the third case is it possible to neglect the role of the slow atoms in the determination of the structure of the Lamb dip. In the first case, on the other hand, their contribution is predominant. Finally, in the second case the effective velocity interval is larger than $\Gamma\ell$ but much smaller than $\bar{v}$.

Thus, depending on the weight with which the slow atoms enter in the distribution of the velocities u, they can make a predominat, a noticeable, or a negligible contribution to the form of the Lamb dip.

Let us consider four types of field configurations of the standing light wave: 1) the light wave is not bounded in all directions (the well-known "homogeneous" problem): 2) the one-dimensional problem - limitation only in the x direction, 3) the two-dimensional problem - x and y are bounded; 4) the three-dimensional problem - x, y, and z bounded, but the resonator length is much larger than the wavelength $\lambda = 2\pi/k$. In case 2) the averaging over u must obviously be made with the distribution (6), while in cases 3) and 4) the distribution (6') must be used.

Let us write out the formulas for the amplitude of the steady-state field near the generation threshold ($\Gamma = \Gamma_m = \Gamma_n$; $\Omega\,\bar{\tau} \ll 1$):

$$G^2/\Gamma^2 = 2(\eta-1)\left(1+\frac{\Gamma^2}{\Gamma^2+\Omega^2}\right)^{-1} - \text{"homogeneous problem,} \tag{7}$$

$$G^2\bar{\tau}/\Gamma = 2\sqrt{\pi}(\eta-1)\left(1+\frac{arctg\,(\Omega/\Gamma)}{\Omega/\Gamma}\right)^{-1} - \text{one-dimensional problem,} \tag{8}$$

$$G^2\bar{\tau}^2 = \frac{8(\eta-1)}{\ln\dfrac{1}{\Gamma\bar{\tau}}}\left(1+\frac{\ln\Gamma\bar{\tau}\,\sqrt{1+\Omega^2/\Gamma^2}}{\ln\Gamma\bar{\tau}}\right)^{-1} - \text{two- and three-dimensional problem.} \tag{9}$$

Here η is the excess of excitation above threshold.

In the one-dimensional case, just as in the "homogeneous" one, the width of the dip is determined by Γ, and in the two- and three-dimensional cases it is determined by $(\Gamma\bar{\tau})^{1/2}$, and not by $1/\bar{\tau}$ as in beam systems. When $\Gamma\bar{\tau} \ll 1$ we obviously have $(\Gamma/\bar{\tau})^{1/2} \ll 1/\bar{\tau}$.

The reliability of the "matching" of the resonator to the transition frequency ω_{mn} is determined by the curvature of the dip at its center. Near the center of the dip, i.e., when $\Omega/\Gamma \ll 1$, we have respectively

$$G^2 \sim 1+\frac{\Omega^2}{2\Gamma^2}, \tag{10}$$

$$G^2 \sim 1+\frac{\Omega^2}{6\Gamma^2}, \tag{11}$$

$$G^2 \sim 1 + \frac{\Omega^2}{4\Gamma^2 \ln (1/\Gamma\bar{\tau})}. \qquad (12)$$

In (10) and (11), the second derivative is proportional to Γ^2, i.e., it is inversely proproportional to the square of the width of the dip. In the two-dimensional and three-dimensional cases, which are of practical importance, the second derivative equals $(2\Gamma^2 \ln 1/\Gamma\bar{\tau})^{-1}$ whereas the reciprocal of the half-width squared is $\bar{\tau}/\Gamma$. Their ratio is $(2\Gamma\bar{\tau} \ln 1/\Gamma\bar{\tau})^{-1}$, so that the parameter determining the reliability of the frequency stabilization may be much smaller than the width of the dip and is determined, in practice, by the level width Γ.

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KINETIC EQUATION FOR A GAS OF SOLITONS

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Many problems describing the propagation of nonlinear waves in dispersive media reduce to the Korteveg - de Vries equation [1]

$$v_t + v_x + vv_x + v_{xxx} = 0 \qquad (1)$$

The existing algorithm [2, 3] for the construction of solutions of (1) fail in practice in those cases when the initial profile v(x, 0) has a complicated form and leads to the formation of a large number of solitary waves (solitons):

$$v = 3\alpha \operatorname{sech}^2 [\sqrt{\alpha}(x - ut)/2], \quad \alpha = u - 1 > 0, \qquad (2)$$

where u is the wave velocity, and we assume that $\alpha \ll 1$. At the same time, such a situation is characteristic of a turbulent medium, and its investigation is of appreciable interest. We develop below a statistical method for the investigation of (1), leading to the possible use of the concept of a Boltzmann "gas" of solitons.

The Hamiltonian formalism for (1) is of the form

$$H \frac{1}{2} \int dq\,(1-q^2)\, v(q)\, v(-q) + \frac{1}{6} \int dq_1\, dq_2\, dq_3\, v(q_1)\, v(q_2)\, v(q_3)\, \delta(q_1+q_2+q_3)$$

$$v = \int dq\, e^{iqx}\, v(q); \quad v(-q) = v^*(q); \quad \dot{v}(q) = iq \frac{\delta H}{\delta v(-q)}. \qquad (3)$$

We consider a "gas" made up of a large number of solitons, the distance between which is on the average much larger than the characteristic width of the solitons $\alpha^{-1/2}$. We represent H in the form