[1] C. Gardner, Annual Report, Princeton MATT-Q-24, 329 (1966).
[2] Yu. A. Berezin and V. I. Karpman, Zh. Eksp. Teor. Fiz. 51, 1557 (1966) [Sov. Phys.-JETP 24, 1049 (1967)].
[3] C. Gardner, J. Greene, M. Kruskal, and R. Miura, Phys. Rev. Lett. 19, 1095 (1967).
[4] I. M. Lifshitz, Usp. Fiz. Nauk 83, 617 (1964) [Sov. Phys.-Usp. 7, 549 (1965)].
[5] B. I. Halperin, Phys. Rev. 139, 194A,(1965).
[6] J. Zittartz and J. S. Langer, Phys. Rev. 148, 741 (1966).

APPROXIMATE SOLUTION OF THE THREE-BODY PROBLEM WITH A LOCAL POTENTIAL

B. Akhmatkhodzhaev[1], V. B. Belyaev, and E. Wrzecionko[2]
Joint Institute for a Nuclear Research
Submitted 5 May 1969
ZhETF Pis. Red. 9, No. 12, 692 - 694 (20 June 1969)

As is well known, to solve the Faddeev equations it is necessary to know the behavior of the two-particle T-matrix off the mass shell. We shall point out one possibility of constructing such a T-matrix. Assume that we have a local short-range potential V(r). The ℓ-th harmonic of the Fourier transform of this potential is given by

$$V_\ell(k,k') = \frac{1}{\pi^2}\int_0^\infty j_\ell(kr)\, j_\ell(k'r)\, V(r) r^2 dr \tag{1}$$

We shall approximate the local potential V_ℓ (k, k') by the aggregate of nonlocal potentials, using the Bateman method [1]. We obtain the following expression for the approximating potential V_ℓ (k, k'):

$$\tilde{V}_\ell(k,k') = T_r[\theta(k,k')\, d^{-1}]. \tag{2}$$

The solution of the Lippman-Schwinger equation with potential V_ℓ (k, k') is

$$T_\ell(k,k',z) = Tr[C(z)\,\theta(k,k')]$$

where

$$\theta_{ij}(k,k') \equiv V_\ell(k,s_i)\, V_\ell(k',s_j),$$

$$d_{ij} \equiv V_\ell(s_i,s_j),$$

$$I_{ij}(z) \equiv \int_0^\infty k^2 dk\, \frac{V_\ell(k,s_i)\, V_\ell(k,s_j)}{k^2 - \sqrt{2\mu_{12} z} - i\epsilon}\,, \tag{3}$$

$$C_{ij}(z) \equiv [(d + 8\pi\mu_{12} I)^{-1}]_{ij},$$

s_i are parameters, i, j = 1, ..., n, and μ_{12} is the reduced mass of the repelling particles.

It is seen from (2) that when k and k' equals one of the parameters s_i, the approximate potential $\tilde{V}_\ell$(k, k') coincides with the local potential (1). It is clear that if the points s_i are uniformly distributed along the axes k and k' and if $n \to \infty$, the approximate potential $\tilde{V}_\ell$(k, k') approaches the local potential (1). For most of the short-range potentials used in the calculations, we can confine ourselves to values of n that do not differ strongly from unity. This is possible as a result of the smoothness of the function V_ℓ(k, k') with respect to the variables k and k'. From the condition that the integral

[1] Inst. of Nuc. Phys., Uzbek Acad. Sci., Tashkent.
[2] Institute for Nuclear Research, Warsaw.

$$\int_0^\infty \left| V(k,k') - \tilde{V}(k,k') \right| dk'$$

be small and the integral of the resolvent of the Lippman-Schwinger equation must be bounded, it follows that the T-matrix (3) differs little from the solution of the Lippman-Schwinger equation with the local potential V(k, k'). Thus, the T-matrix (3) has the correct behavior both on and off the mass shell. The smaller the coupling constant, the better this estimate of the approximate T-matrix (3). It follows therefore that a calculation of the binding energy of the 3-body system with T-matrix (3) will be the more accurate, the smaller the coupling constant. This is indeed the situation observed in [3] and in the present work.

GM/μ \ n	1	2	3	4
1,6	0,25	0,2621	0,3015	0,348
2,0	0,33	0,587	0,6798	0,708
2,4	0,41	0,8666	1,0376	1,086
2,8	0,49	1,119	1,385	1,449
4,0	0,73	1,689	2,389	2,468

The T-matrix (3) with n = 1, 2, 3, and 4 was used to solve the Faddeev equations for a system of three spinless particles interacting via a Yukawa potential $V(r) = G[e^{-\mu r}/r]$. The higher configurations in the relative two-particle motion were disregarded, since their contribution, as estimated in [2], is negligibly small. We calculated the dependence of the binding energy of the system on the coupling constant G. The parameters of the potential were chosen to be the same as in [3]. Just as in the calculations with a local potential [3], three levels appear for n = 2, 3 and 4. The table lists the values of $\alpha\ [|E|M/\mu^2]^{1/2}$ for the ground state.

The authors thank Professor L. D. Faddeev for a stimulating discussion.

[1] H. Bateman, Proc. Roy. Soc. A, 100, 441 (1922).
[2] V. B. Belyaev and W. Wrzecionko, JINR Preprint R4-4144 (1968).
[3] J. W. Humbertson, R. L. Hall, and T. A. Osborn, Phys. Lett. 27B, 195 (1968).
[4] J. S. Ball and D. Y. Wong, Phys. Rev. 169, 1362 (1968).
[5] L. V. Kantorovich and V. I. Kryolov, Priblizhennye metody vysshego analiza (Approximate Methods of Higher Analysis), Gostekhizdat, 1950.