CONCERNING CERTAIN CONSEQUENCES OF THE REGGE INTERACTION OF HADRONS IN THE QUARK MODEL

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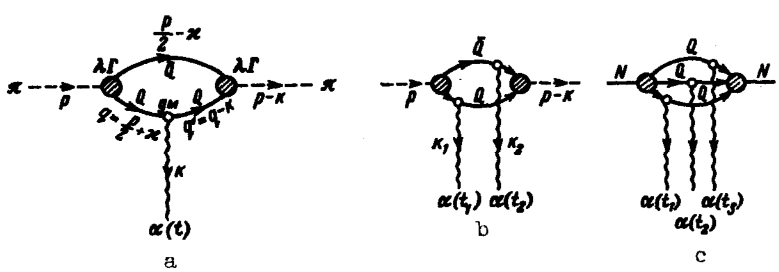
We investigate the dependence of the residue r(t) of the Pomeranchuk pole on the momentum-squared t in the quark model. We shall show that in this model, at high energies and low values of t, the dependence of the residue on t is determined in the main by the function F(t), where F is the electromagnetic form factor of the hadron. An analogous investigation was made for the amplitudes $N_2(t, t_1, t_2)$ and $N_3(t, t_1, t_2, t_3)$ for the production of two reggeons on the hadron (t_1, t_2, and t_2 are the squares of the reggeon momenta).

Hadron scattering at high energies is described by the exchange of reggeons between the quarks making up the hadrons (in analogy with the case of hadron scattering by the component nuclei of the hadron, considered by Galuber [1]). If we confine ourselves to contributions of non-enhanced diagrams [2], then the single-, two-, and three-reggeon contributions to various hadron-hadron scattering processes are determined by the functions r, N_2, and N_3, corresponding to the typical diagrams shown in the figure. In the nonrelativistic quark model, these diagrams make at small t, t_1, t_2, and t_3 the predominant contribution to the functions r, N_2, and N_3.



Let us consider by way of the simplest example the $\pi\pi$ scattering process. The residue r(t) is determined then by the diagram of Fig. a. Here $K^2 = t$, Q and $\bar{Q}$ are the quark and antiquark, regarded as weakly-noninteracting nonrelativistic Dirac particles [3, 4] with effective mass M and with a small relative momentum κ satisfying the conditions

$$\kappa^2 << m^2, \quad \vec{\kappa}^2 \sim \Delta^2 = M^2 - \frac{m^2}{4} << M^2, \quad \kappa_0 \sim \frac{\Delta^2}{m} \tag{1}$$

(m is the pion mass) and with a residue g_μ given by [4]:

$$g_\mu(q, q', t) = (q + q')_\mu [g_0(t) + g_1(t)] + \frac{t - 4M^2}{2M} g_1(t)\gamma_\mu, \tag{2}$$

where g_0 and g_1 coincide, apart from a kinematic factor, with the residues ρ_0 and ρ_1 introduced in [5].

The contribution of the Pomeranchuk pole $\alpha(t)$ to the $\pi\pi$ scattering amplitude is equal to

$$A_1(s,t) = sr^2(t)\,G_\alpha(s,t) \tag{3}$$

where s is the square of the pion energy and $G_\alpha(s, t) = \sigma_\alpha(s/s_0)^{\alpha(t)-1}$, where σ_α is the signature factor and s_0 is a scale factor that can be set equal to m^2. On the other hand,

$$A_1(s,t) = r_\mu(p_1, k)\, r_\mu(p_2, -k)\, G'_\alpha(s', t),$$

where $r_\mu(p, k)$ is the vertex part shown in Fig. a, p_1 and p_2 are the 4-momenta of the repelling pions, s' is the square of the energy of the reggeon-exchanging quarks,

$$G'_\alpha(s', t) = \sigma_\alpha\left(\frac{s'}{s'_0}\right)^{\alpha(t)-1}, \tag{4}$$

and $s'_0 = M^2$. When conditions (1) are satisfied, $s' \approx s/4$ and $s_0 \approx 4s'_0$, we obtain $G'_\alpha(s', t) \approx G_\alpha(s, t)$ and $r_\mu(p_1, k)r_\mu(p_1, k)r_\mu(p_2, -k) = sr^2(t)$. Calculating $r_\mu(p, k)$ in accordance with Fig. a we can show that in the high-energy limit

$$r_\mu(p, k) = \sqrt{2}\, p_\mu r(t),$$

with

$$r(t) = i\sqrt{2}\, i\alpha\lambda^2 \int \frac{d^4\kappa}{(2\pi)^4} \frac{[(2m^2 - t)\, g_0(q^2, q'^2, t) + t g_1(q^2, q'^2, t)]\Gamma(\kappa^2)\Gamma[(\kappa - \frac{k}{2})^2]}{[(\frac{p}{2} + \kappa)^2 - M^2 + i\epsilon][(\frac{p}{2} - \kappa)^2 - M^2 + i\epsilon][(\frac{p}{2} + \kappa - k)^2 - M^2 + i\epsilon]}. \tag{5}$$

It is easy to verify that when conditions (1) are satisfied and when $(t) << m^2$ formula (5) yields (the functions $g_{0,1}$ are taken outside the integral sign)

$$r(t) = \frac{m}{\sqrt{2}}\{g_0(t) + \frac{t}{2m^2}[g_1(t) - g_0(t)]\}\, S\left(\frac{k^2}{4}\right), \tag{6}$$

where

$$S\left(\frac{k^2}{4}\right) = m\lambda^2 \int \frac{d^3\kappa}{(2\pi)^3} \frac{\Gamma(\vec{\kappa}^2)\,\Gamma[(\vec{\kappa} - \frac{\mathbf{k}}{2})^2]}{(\Delta^2 - \vec{\kappa}^2)\,[\Delta^2 - (\vec{\kappa} - \frac{\mathbf{k}}{2})^2]} \tag{7}$$

is the pion form factor, determined by the spatial distribution of the quarks in it. Assuming, as usual [3, 4], that the quark radius is much smaller than the hadron radius, we can state, first, that the coefficient preceding $S(k^2/4)$ in (6) depends little on t, and second that the form factor $S(k^2/4)$ equals the hadron electromagnetic from factor $F(k^2)$.

We have calculated in similar fashion the contribution of diagram b to the function $N_2(t, t_1, t_2)$:

$$N_2(t, t_1, t_2) = \Phi(t, t_1, t_2)\, F(2t_1 + 2t_2 - t), \tag{8}$$

where Φ is a function that depends relatively little on its argument (it is qudratic in g_0 and g_1). When $\vec{k} = \vec{k}_1 = 0$ we have

$$N_2(0, 0, 0) = \frac{m^2}{2\sqrt{2}} g_o^2(0) = \frac{r^2(0)}{\sqrt{2}}, \qquad (9)$$

which coincides with the lower bound of $N_2(0, 0, 0)$ obtained in [7].

It can be assumed that the results are valid not only when (t), (t_1), (t_2) << m^2, but also in a wider range. The basis for such an assumption may be the circumstance that the relations for the Sachs electromagnetic form factors of the nucleons, obtained in [3], are valid up to $|t| \sim 2\ (\mathrm{GeV}/c)^2$. Yet these relations should be violated as a result of the same factors that limit our assumptions. At large t the function F changes much more strongly than the factors contained in (6) and (8), and these formulas reduce to the statements that $r(t) \propto F(t)$ and $N_2(t) \propto F(2t_1 + 2t_2 - t)$. It is clear that a formula of the form $r(t) \propto F(t)$ is valid not only for the pion, but also for any hadron. However, the expression for N_2 depends on the number of quarks making up the hadron under consideration. For example, for a nucleon made up of three quarks, an examination of Fig. c without the $\alpha(t_3)$ line, corresponding to the emission of two reggeons, yields

$$N_2(t, t_1, t_2) \propto F[\frac{3}{2}(t_1 + t_2) - \frac{1}{2}t]. \qquad (10)$$

In analogy, an analysis of the entire diagram c yields

$$N_3(t, t_1, t_2, t_3) \propto F[\frac{3}{2}(t_1 + t_2 + t_3) - \frac{1}{2}t].$$

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