THE SIGN OF THE PRODUCT OF THE REDUCED MATRIX ELEMENTS OF E2 TRANSITIONS BETWEEN THE 0^+, 2^+, AND 2_2^+ STATES IN EVEN-EVEN NUCLEI

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Submitted 8 May 1969
ZhETF Pis. Red. 9, No. 12, 698 - 702 (20 June 1969)

A number of recent papers are devoted to the effect or reorientation in even-even nuclei, and deal with the measurement of the static quadrupole moments ($Q_{2_1^+}$) of the first levels with characteristic 2_1^+.

The determination of the values of $Q_{2_1^+}$ is complicated by the contribution made to the total cross section for the Coulomb excitation of the 2_1^+ level by the interference terms connected with the virtual excitation of the higher levels. This contribution is especially pronounced in the case of virtual excitation of the 2_2^+ level. The magnitude of this contribution can be calculated if one knows the value of the product (Π) of the three reduced matrix elements $M_{r,s}$(r, s - states with characteristics 0^+, 2_1^+, and 2_2^+):

$$\Pi = \langle 0 \| m(E2) \| 2_1 \rangle \langle 0 \| m(E2) \| 2_2 \rangle \langle 2_2 \| m(E2) \| 2_1 \rangle. \tag{1}$$

In experiments on the Coulomb excitation of the 2_1^+ and 2_2^+ levels one determines the values of the reduced probabilities B(E2: r → s). Using the relation

$$M_{rs}^2 = (2I_r + 1) B(E2; r \rightarrow s) \tag{2}$$

it is possible to determine the magnitude (but not the sign) of Π from the experimental data. The impossibility of determining the sign of Π makes the values of $Q_{2_1^+}$ obtained in experiments on the orientation to be ambiguous. In a number of cases the uncertainty of the sign of Π makes the discrepancy between the values of $Q_{2_1^+}$ very large. Thus, in [1], the values of $Q_{2_1^+}$ for the ^{148}Sm nucleus is either -0.58 ± 0.01, or -0.40 ± 0.01, depending on the sign of Π. According to [2] the value of $Q_{2_1^+}$ for the ^{144}Nd nucleus is, depending on the sign of Π, either -0.93 ± 0.17 or -0.32 ± 0.30. It is therefore very important to estimate the sign of Π on the basis of theoretical considerations. Tamura[3] found for the particular case of ^{114}Cd that Π is positive. This result was obtained by starting from a description of the 2_2^+ state as a mixture of single- and two-phonon states. The ratio of the amplitudes of the mixed functions and their relative phase were taken in[3] from data obtained for the 2_2^+ state in an investigation of the differential cross section of the reaction ^{114}CD(p, p')^{114}Cd*.

Within the framework of Davydov's phenomenological theory [4], the question of the sign of Π can be solved uniquely and in a unified manner for all even-even nuclei, without resorting to additional data obtained by investigating the nuclear reactions.

In that theory, which takes into account the connection of the rotational motion with oscillations of the nuclear surface, the expression for the E2-transition operator is written in the form

$$m(E2, \mu) = Q_0 \frac{\beta}{\beta_0} q_{2\mu} \sqrt{5/16\pi}, \tag{3}$$

where

$$Q_0 = \frac{3Ze R_0^2 \beta_0}{\sqrt{5\pi}}; \quad q_{2\mu} = D^2_{\mu 0}(\theta_i) + \{D^2_{\mu 2}(\theta_i) + D^2_{\mu -2}(\theta_i)\} \frac{\sin\gamma_0}{\sqrt{2}}. \tag{4}$$

The wave functions are given by

$$|IM_{\tau\nu}> = \phi_{I\tau\nu}(\beta)\, \Phi_{IM\tau}(\theta_i) \tag{5}$$

with

$$\Phi_{IM\tau}(\theta_i) = \sum_{K=0,2\ldots I} |IMK> A^{\tau}_{IK} \tag{6}$$

here $|IMK>$ are the symmetrical-top functions, A_{IK} are coefficients that depend on effective phenomenological value γ_0 of the dynamic variable γ (see [4]), and $\phi_{I\tau\nu}$ is a function of the dynamic variable β characterizing the β-oscillations. The index $\tau = 1, 2, \ldots n_1$ numbers in increasing order the energies of the excited states with spin I in the rigid asymmetrical rotator, ν are the roots of a transcendental equation (see [4]) numbered in order of increasing magnitude, and β_0 is the equilibrium deformation; β, $\beta_0 \geq 0$.

The reduced matrix elements that enter in H are given by the expression

$$<I'M'\tau'\nu'|m(E2,\mu)|IM\tau\nu> = (-1)^{I'-M'} \begin{pmatrix} I' & 2 & I \\ -M & \mu & M \end{pmatrix} \times$$
$$\times <I'\tau'\nu'||m(E2)||I\tau\nu>, \tag{7}$$

where the indices $I\tau\nu$ for the states 0^+, 2^+_1, and 2^+_2 are respectively $01\nu_0$, $21\nu_0$, and $22\nu_0$. From (3), (4) and (5) it follows that

$$<I'\tau'\nu'||m(E2)||I\tau\nu> = Q_0 S_{I'\tau'\nu',I\tau\nu} F_{I'\tau',I\tau} \sqrt{5/16\pi}, \tag{8}$$

where

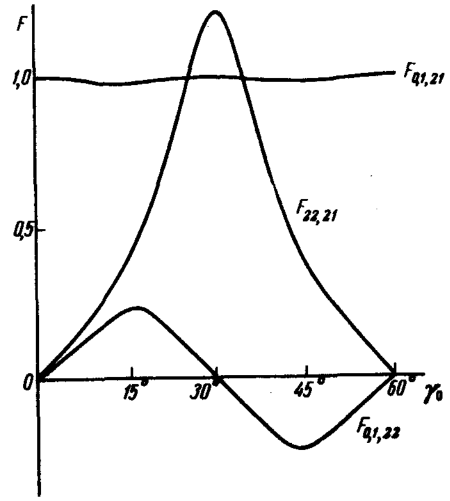
$$F_{I'\tau',I\tau} = \sum_{K,K'} <I'K'||q_2||IK> A^{\tau}_{IK} A^{\tau'}_{I'K'}, \tag{9}$$

and

$$S_{I'\tau'\nu',I\tau\nu} = \int_0^\infty \phi_{I'\tau'\nu'}(\beta) \frac{\beta}{\beta_0} \phi_{I\tau\nu}(\beta)\, d\beta. \tag{10}$$

We present below the values of F for E2 transitions between the states 0^+, 2^+_1, and 2^+_2 of interest to us:

$$F_{01,21} = (A^1_{20}\cos\gamma_0 + A^1_{22}\sin\gamma_0)A^1_{00},$$
$$F_{01,22} = (A^2_{20}\cos\gamma_0 + A^2_{22}\sin\gamma_0)A^1_{00}, \tag{11}$$
$$F_{22,21} = \sqrt{10/7}\{(A^2_{22}A^1_{22} - A^2_{20}A^1_{20})\cos\gamma_0 + (A^2_{22}A^1_{20} + A^2_{20}A^1_{22})\sin\gamma_0\}.$$



The numerical values of the matrix elements are shown in the figure as functions of γ_0. It is seen from the figure that the product of the matrix elements F contained in Π is always positive when $0° < \gamma_0 < 30°$ and negative when $30° < \gamma_0 < 60°$. Since each of the wave functions enters twice in the products under consideration, the determined sign of the product of the matrix elements does not depend on the choice of the phases of the wave functions Φ. We consider now the signs of the matrix elements S. When the non-adiabaticity parameter (μ) tends to zero we have $S \to +1$. It is known from [5] that the values of S^2 increase monotonically with increasing μ. Consequently, the matrix elements remain positive at finite μ.

Thus, Π is always positive when $\gamma_0 < 30°$ and negative when $\gamma_0 > 30°$. In the theory under consideration

$$Q_{2_1^+} = -Q_0 \frac{6\cos(3\gamma_0)}{7\sqrt{9-8\sin^2(3\gamma_0)}} S_{21\nu_0,21\nu_0}. \qquad (12)$$

we see that $Q_{2_1^+} < 0$ when $\gamma_0 < 30°$ and $Q_{2_1^+} > 0$ when $\gamma_0 > 30°$.

We thus conclude from Davydov's theory that in all cases when the experimental values of $Q_{2_1^+}$ are negative, regardless of the sign of Π, the correct value of $Q_{2_1^+}$ is that corresponding to positive Π; conversely, if $Q_{2_1^+}$ is positive for any sign of Π, the correct value of $Q_{2_1^+}$ corresponds to negative Π.

Our estimate of the sign of Π for the ^{114}Cd nucleus (in which case $Q_{2_1^+}$ is negative) coincides with the result of [3]. We know of only one experimental paper, and a preliminary one at that, in which the sign of Π has been determined for ^{114}Cd. Its result does not agree with the theoretical estimates performed in either that paper or in [3].

Experiments to determine the sign of Π, first are of appreciable interest for the verification of the aforementioned conclusion of the theory, and second, if confirmed, will make it possible to eliminate in many cases the ambiguity in the values of $Q_{2_1^+}$ obtained in reorientation experiments.

The authors are grateful to A. S. Davydov and V. I. Ovcharenko for discussions.

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BRANCHING RATIO OF $K \to 3\pi$ DECAYS AND DETERMINATION OF PION SCATTERING LENGTHS

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Submitted 14 May 1969
ZhETF Pis. Red. 9, No. 12, 702 - 705

The experimental values of the probabilities of $K \to 3\pi$ decays is well described by formulas of the type

$$A + B\left(T - \frac{1}{2}T_{max}\right), \tag{1}$$

where T is the kinetic energy of one of the pions, and A and B are certain constants. The total decay probability R is determined by the value of A, for the second term drops out upon integration over the phase volume. The $\Delta I = 1/2$ rule relates the constants A for different decays, and three relations exist between the total probabilities of the transitions $K^+ \to \pi^+\pi^+\pi^-$, $\pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$. It is customary to compare the decay probabilities R divided by the phase volume Φ, which depends strongly on the pion and kaon mass differences as a result of the smallness of the energy released in the decay. It turns out, however, that the influence of the mass difference is not excluded completely in such relations [1 - 4]. As a result of the mass differences in different decays, changes take place in the position of the physical region and in the value of T_{max}. The decay probabilities depend strongly on the pion energies (B is large), and therefore a shift of the physical region leads to an appreciable change (up to 10%) of the decay probability. These effects cannot be taken into account in explicit form, since they depend strongly on the form in which the matrix element has been written out, and on the choice of the variables describing the pion spectra. Only the relation

$$\frac{4R^{00+}}{R^{++-}} - \frac{2R^{000}}{3R^{+-0}} = 0 \tag{2}$$

is independent of the mass differences. This relation can be written both for the probabilities R divided by the phase volume and for the R themselves, since effects connected with the change of the phase volume also cancel out in (2). Relation (2) is violated only by transitions with $\Delta I = 7/2$, and remains unchanged for transitions with $\Delta I = 1/2$, $3/2$, and $5/2$.

The interaction of the produced pions violates the linear character of (1). For