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BRANCHING RATIO OF $K \to 3\pi$ DECAYS AND DETERMINATION OF PION SCATTERING LENGTHS

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Submitted 14 May 1969
ZhETF Pis. Red. 9, No. 12, 702 - 705

The experimental values of the probabilities of $K \to 3\pi$ decays is well described by formulas of the type

$$A + B\left(T - \frac{1}{2}T_{max}\right), \qquad (1)$$

where T is the kinetic energy of one of the pions, and A and B are certain constants. The total decay probability R is determined by the value of A, for the second term drops out upon integration over the phase volume. The $\Delta I = 1/2$ rule relates the constants A for different decays, and three relations exist between the total probabilities of the transitions $K^+ \to \pi^+\pi^+\pi^-$, $\pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$. It is customary to compare the decay probabilities R divided by the phase volume Φ, which depends strongly on the pion and kaon mass differences as a result of the smallness of the energy released in the decay. It turns out, however, that the influence of the mass difference is not excluded completely in such relations [1 - 4]. As a result of the mass differences in different decays, changes take place in the position of the physical region and in the value of T_{max}. The decay probabilities depend strongly on the pion energies (B is large), and therefore a shift of the physical region leads to an appreciable change (up to 10%) of the decay probability. These effects cannot be taken into account in explicit form, since they depend strongly on the form in which the matrix element has been written out, and on the choice of the variables describing the pion spectra. Only the relation

$$\frac{4R^{00+}}{R^{++-}} - \frac{2R^{000}}{3R^{+-0}} = 0 \qquad (2)$$

is independent of the mass differences. This relation can be written both for the probabilities R divided by the phase volume and for the R themselves, since effects connected with the change of the phase volume also cancel out in (2). Relation (2) is violated only by transitions with $\Delta I = 7/2$, and remains unchanged for transitions with $\Delta I = 1/2$, 3/2, and 5/2.

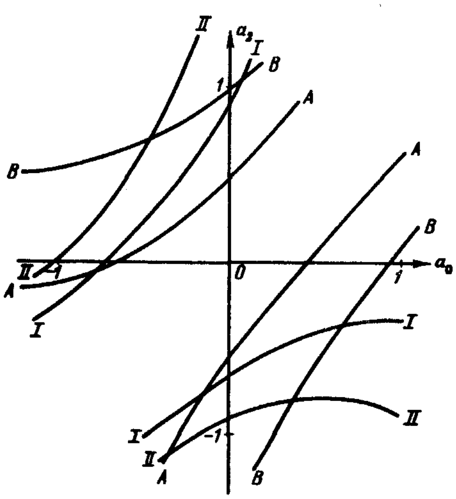
The interaction of the produced pions violates the linear character of (1). For three-particle reactions with small energy release, it is possible to develop a rigorous phenomenological theory analogous to the Bethe-Peierls theory for two-particle reactions. Within the framework of this theory, we can take into account the interaction of the pions produced in the $K \to 3\pi$ decay [5, 6]. It turns out that when $|a_I| \leq 1$ (a_I - scattering lengths of pions with isospin I in units of $\hbar/m_\pi c$) the pion interaction does not alter in noticeable fashion the linear formula (1) in the $K^+ \to \pi^+\pi^+\pi^+$ decay, viz., when $T \geq 0.9T_{max}$ the deviation from the linear form is of the order of $0.02(a_0 - a_2)^2 A$. The accuracy of the existing experimental data is insufficient to discern such deviations, the observation of which calls for the analysis 10^5 - 2×10^5 events. The greatest deviation from nonlinearity (larger by almost one order of magnitude that in $K \to \pi^+\pi^+\pi^-$) occur in the decays $K \to \pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$. Data on these decays, obtained by various workers (about 6000 events) are gathered in [7]. The pion spectra deviated noticeably from linearity. A comparison of these spectra with the theoretical formulas yielded the possible values of the pion scattering lengths. In the figure, they lie between curves I and II.

Violation of the linear character of (1) gives rise in the right side of (2) to an expression that can be expanded in a series in $a_I E^{1/2}$ (E is the energy released in the decay). When the first two terms of the series are included, formula (3) takes the form

$$\frac{4R^{00+}}{R^{++-}} - \frac{2R^{000}}{3R^{+-0}} = \frac{175}{162}\left(1 - \frac{3\sqrt{3}}{2\pi}\right)(a_0 - a_2)^2 m_\pi E - \frac{160\sqrt{3}}{729\pi}(a_0 - a_2)^2(2a_0 + a_2)(m_\pi E)^{3/2} .$$

Recently published experimental data [8] make it possible to determine the left side of (3):

$$\frac{R^{00+}}{R^{++-}} = 0,305 + 0,008, \quad \frac{R^{000}}{R^{+-0}} = 1,696 \pm 0,074$$



Two strips between curves A and B - possible values of the pion scattering lengths (in units of $h/m_\pi c$), obtained from a comparison of (3) with the experimental data; the two strips between curves I and II are the scattering lengths obtained from the deviations of the pion spectra from linearity in the decays $K^+ \to \pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$ [7].

If the transitions with $\Delta I = 7/2$ are small (this assumption is quite reasonable), then (3) makes it possible to estimate the pion scattering lengths. The values obtained in this manner are shown in the figure. The right side of (3) is determined mainly by the deviations from the linear form (1) in the decays $K \to \pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$, so that we can expect beforehand the lengths determined from (3) and from the pion spectra [7] to lie more or less in the same region. It is seen from the figure that a considerable overlap of the regions does indeed take place. This confirms the consistency of the different experimental data, and consequently also of the obtained possible values of the pion scattering lengths.

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ELECTRIC FORM FACTOR OF THE PROTON AT SMALL MOMENTUM TRANSFERS

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Submitted 4 June 1969

ZhETF Pis. Red. 9, No. 12, 705 - 707 (20 June 1969)

Experimental data on the behavior of the proton form factor at small momentum transfers are quite inaccurate [1]. For the electric form factor of the proton $G(q^2)$, the errors in the measurment of the function

$$\phi(q^2) = \frac{1 - G(q^2)}{q^2}; \tag{1}$$

are comparable with the values of the function itself at values $q^2 < 1\ F^{-2}$ ($F = 10^{-13}$ cm), and there are no data below $q^2 = 0.2\ F^{-2}$. It is customarily assumed that in the region of small transfers the form factor is described, with a high degree of accuracy, by the relation $\phi(q^2) = 1/6\ r_0^2$, where $r_0 = 0.8$ F is the mean-square radius of the proton, obtained by extrapolation from the region $q^2 > 1\ F^{-2}$. However, the experimental data leave room for various hypotheses concerning the variation of the behavior of the form factor at small values of q^2. By way of an example, we indicate the hypothetical proton "halo" [2], which was introduced to explain the discrepancies between theory and experiment in the Lamb shift [3] and in spectra of heavy muonic atoms [2].

We have shown in [4] that the same discrepancies can be explained by assuming a small nonlinearity of the equations of electrodynamics. In the case of interest to us, when only the electric field E is significant, this assumption reduces to the following change of the field Lagrangian