We note that the logarithmic terms of the form factor appear in the nonlinear theory only for charged particles. There should be no such terms for the neutron, in accord with experiment, since measurements of the mean-square radius of the neutron at high energies and in the scattering of slow neutrons by electrons which do not contradict each other [6].

In conclusion, we emphasize once more the importance of measuring the proton form factor at low transfers; such measurements can be made with strong-current electron accelerators with energies 10 = 100 MeV.

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CONCERNING THE ORIGIN OF SUPERHEAVY ELEMENTS

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Recently, P. Fowler's group (private communication) observed in the cosmic ray spectrum nuclei with charge $Z \approx 108 \pm 2$. According to the notions prevailing to date [1, 2], such heavy elements cannot result from the r-process occurring in supernova flares. The reason is that the r process is terminated as a result of the neutron-induced nuclear fission, which occurs, according to the latest estimates of F. Hoyle and W. Fowler [3], at mass numbers $A \approx 270 - 275$ $(Z \approx 93)$. These estimates were made with allowance for the influence of the symmetry term $[(N - Z)/A]^2$ in the surface energy of the nuclear drop on the fissility, but without allowance for shell corrections [4,5] in the mass formula; these corrections become particularly large in the region of magic nucleon numbers, and alter appreciably both the height and the width of the barrier [6].

Inasmuch as the termination of the r-process occurs, according to the estimates of [3], when the number of neutrons is close to the magnic $N = 184$, allowance for the shell corrections in the calculation of the course of the r-process becomes essential.

The path of the r-process can be calculated by assuming statistical equilibrium in the $(n\gamma)$ and (γn) reactions; this leads to the condition [1]

$$B_n(Z, N) = T_9 / 5{,}04\,(34{,}07 - \log n_n + 1{,}5 \log T_9), \tag{1}$$

under which the capture of neutrons by the nucleus (Z, N) stops. In formula (1).

$B_n(Z, N)$ denotes the binding energy of the external neutron in the nucleus (Z, N), n_n is the numerical density of the neutrons (in units of cm^{-3}) captured by the nuclei during the supernova flare, and T_9 is the temperature (in units of 10^9 °K) at which this process is realized.

Figure 1 shows a fragment of the path of the r-process, calculated by us for various values of B_n from the mass formulas of Myers and Swiatecki [4]. As shown by an analysis of [7], these formulas can result in good agreement between the calculated and experimental values of the abundance of the elements in the universe, although at different values of the pair T_9 and n_n for different regions of the nuclei. The broken lines 1, 2, and 3 of Fig. 1 correspond to neutron binding energies B_n = 1.6, 2.0, and 2.7 respectively.

In order for the r-process not to terminate and to be able to proceed further, it is necessary to satisfy simultaneously the following conditions (see Fig. 2):

$$\lambda_{\beta}(Z, N) \gtrsim \lambda_{SF}(Z, N), \tag{2a}$$

$$\lambda_{n\gamma}(Z+1, N-1) \gtrsim \lambda_{nF}(Z+1, N-1), \tag{2b}$$

$$\lambda_{n\gamma}(Z+1, N-1) \gtrsim \lambda_{\beta}(Z+1, N-1), \tag{2c}$$

where λ_β, $\lambda_{n\gamma}$, λ_{SF}, and λ_{nF} are the probabilities, per unit time, of β decay, the $(n\gamma)$ reaction, spontaneous fission (SF), and neutron fission (nF), respectively.

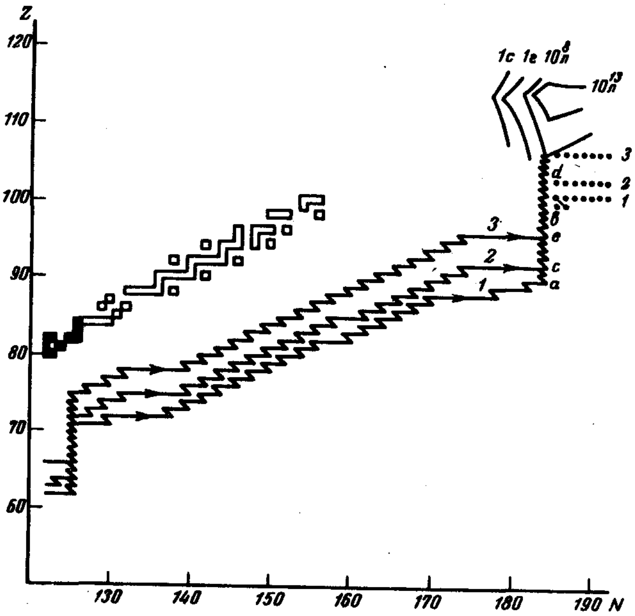

Fig. 1. Paths of r-process for heavy nuclei at various values of the neutron binding energy: ■ - stable nuclei, □ - β-stable nuclei.

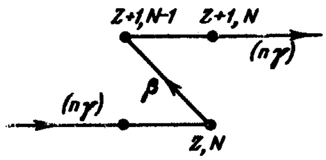

Fig. 2. Fragment of r-process path.

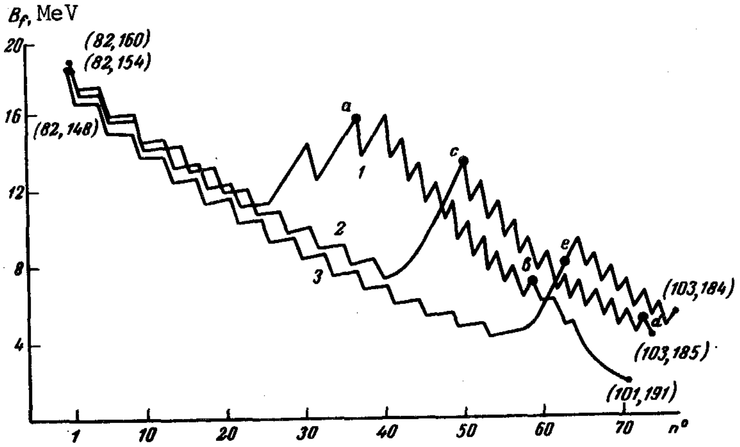

Fig. 3. Fission barriers of nuclei along the r-process path: n^0 - number of nucleus, starting with Z = 82; ab and cd - regions corresponding to the r-process branches with increasing values of Z at N = 183, 184; e corresponds to the value N - 184 for the path 3 with the smallest Z.

The β-decay probability can be estimates from the data [4, 8], using the monograms of [9]; the rate of the (nγ) reaction was calculated from the expression [2]

$$\lambda_{n\gamma} = n_n < \sigma_{n\gamma} v > \simeq 4 \cdot 10^{-17} n_n T_9^{1/2} \text{ (sec}^{-1}\text{)}, \tag{3}$$

where $\sigma_{n\gamma}$ is the cross section of the (nγ) reaction and v is the neutron velocity. The spontaneous and induced fission widths [8, 10] were estimated using the values of the fission barriers, including shell corrections [4].

Figure 3 shows the change of the barriers, starting with Z = 82, in the path of the r-process, for the three cases shown in Fig. 1. We see that in all cases, for nuclei with $N \leq 184$ neutrons, the barriers are sufficiently large (λ_{SF} and $\lambda_{nF} << 1$ sec^{-1}), and since $1 \leq \lambda_\beta \leq 10^2$ sec^{-1} in the region of heavy nuclei with large neutron excesses, the inequality (2a) is always satisfied. The inequality (2c) is also satisfied, as follows from estimates by means of expression (3). For paths 1 and 2 of the r-ptocess (Fig. 1), according to the barrier values (Fig. 3) and expression (3), the inequality (2b) will be satisfied for all values of the pair of conjugate parameters T_9 and n_n satisfying Eq. (1), up to values $N \leq 184$, in the interval $T_9 \approx (1 - 3) \times 10^9$ °K and $n_n \approx 10^{19} - 10^{32}$ cm^{-3}, which are assumed to be the most probable for supernova flares [11]. For the r-process path denoted by the broken line 3, the inequality (2b) will be satisfied only if $n_n \gtrsim 10^{27}$ and, according to (1), $T_9 \gtrsim 1.8$.

The termination of the r-process along paths 1 and 2 occurs when N > 184. According to the mass values of [4], this takes place immediately beyond N = 184. (This part of the path is shown dotted on Fig. 1). However, these values does not take into account the fact that Z = 114 is a magic number, and give underestimated values of the fissure barriers when Z is close to this value.

Allowance for the fact that Z = 114 is magic was made in the papers of S. Nilsson et al. [6, 12], who used in the calculation of the fission barriers the liquid-drop masses of [4] and the shell corrections [5] with allowance for the single-particle structure as in [13]. It has turned out that the barriers are not lower than 5 MeV, in any case for $Z \gtrsim 105$ at values of N from 184 up to 190. Figure 1 shows a topogram of the spontaneous-fission periods, taken from [12].) Therefore, if the r-process develops along line 3, then it reaches $Z = 107$ and $N = 184$, at which the maximum intensity of element production takes place, and continues further at $N > 184$, as a result of which β-stable nuclei with $Z > 107$ can be produced following chains of successive β decays.

If the r-process develops along paths 1 and 2, then the fact that $Z = 114$ is magic is of less importance (especially for path 1), and therefore the r-process may terminate at lower values of N than in the case of path 3. The maximum intensity of formation of β-stable superheavy elements should occur, for paths 1 and 2, at $A \approx 285 - 290$. Since the fission barriers are smaller than the mass differences of the neighboring isobars on the path of β-decay chains, starting from lines 1 and 2 at $N = 184$, the effect of delayed fission [14] may become manifest in the intensity of element formation.

The results are only qualitative, principally because of the inaccurate values of the masses of nuclei with large neutron excesses. In light of the obtained estimates, according to which the r-process, calculated for $B_n \leq 2.7$ MeV, reaches the magic number $N = 184$ and develops at $N > 184$, we can now understand the results of P. Fowler's experiments [1] mentioned at the beginning of the article.

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ERRATA

The reference following the article by V. I. Karpman (Vol. 9, No. 8), on p. 293, should read "... Zh. Eksp. Teor. Fiz. 56, 1952 (1969)."

The footnote of the article by L. D. Derkacheva and A. I. Krymova (Vol. 9, No. 10), on p. 345, should read "Appearance of one spectral line," not"Suppression of one spectral line."