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CONNECTION BETWEEN THE ANTISYMMETRICAL EXCHANGE INTERACTION AND THE CHANGE OF MULTIPLICITY

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Submitted 13 March 1967; resubmitted 20 April 1967
ZhETF Pis'ma 6, No. 6, 697-700 (15 September 1967)

In the theory of weak ferromagnetism [1,2], an important role is played by exchange interaction of the type

$$H_1 = D[S_1 \times S_2], \tag{1}$$

which is antisymmetrical with respect to the spins $\vec{S}_1$ and $\vec{S}_2$. On the other hand, in the theory of exchange interaction connected with the change of multiplicity [3], antisymmetrical spin operators are also significant. This raises the natural question of the connection between the interaction (1) and the change of multiplicity.

To establish this connection, we introduce symmetrical and antisymmetrical spin operators

$$S = S_1 + S_2, \quad A = S_1 - S_2, \tag{2}$$

where $\vec{S}_1$ and $\vec{S}_2$ satisfy the commutation relations:

$$[S_1 \times S_1] = iS_1, \quad [S_2 \times S_2] = iS_2. \tag{3}$$

From (2) and (3) we get the following commutation relations for $\vec{S}$ and $\vec{A}$:

$$[S \times S] = [A \times A] = i S \tag{4}$$

$$S_i A_k - A_k S_i = i e_{ikl} A_l, \tag{5}$$

where e_{ikl} is an antisymmetrical unit tensor of third rank. Relation (5) leads [4] to the equality

$$S^4 A_x - 2S^2 A_x S^2 + A_x S^4 = 2(S^2 A_x + A_x S^2) - 4S_x (S \cdot A), \tag{6}$$

from which it follows that the operator $\vec{A}$ can transform the state of a system with spin S only into a state with spin S' satisfying the condition

$$S' - S = 0, \pm 1 \ . \tag{7}$$

The equality S' - S = 0 (conservation of multiplicity) can occur only if the last term of (6) does not vanish. In our case it follows from (2) that

$$(S \cdot A) = S_1^2 - S_2^2 = S_1(S_1 + 1) - S_2(S_2 + 1). \tag{8}$$

Consequently, if we confine ourselves to the case of two identical spins ($S_1 = S_2$), then

$(\vec{S}\cdot\vec{A}) = 0$ and the operator $\vec{A}$ leads to a change in the multiplicity of the system.

According to (2), we can express $\vec{S}_1$ and $\vec{S}_2$ in terms of $\vec{S}$ and $\vec{A}$ in the form

$$S_1 = \frac{1}{2}(S + A), \quad S_2 = \frac{1}{2}(S - A). \tag{9}$$

Substituting (9) in (1) we obtain, with allowance for (4), the following expression for the antisymmetrical exchange interaction:

$$H_1 = \frac{1}{4} D \{[A \times S] - [S \times A]\}. \tag{10}$$

Another convenient form can be obtained by taking into account relation (5). Applying (5) twice, we get

$$H_1 = \frac{i}{4} S (AS^2 - S^2A). \tag{11}$$

Expression (10) or (11) establishes a connection between the antisymmetrical exchange interaction and the antisymmetrical spin operator of [3]. In the case when $S_1 = S_2$, expression (10) or (11) allows us to state that in a system of two spins of equal magnitude, the antisymmetrical exchange interaction (1) causes a change in the multiplicity of the system.

In the case when the unperturbed ground state is singlet ($S = 0$), the interaction (1) includes, in accordance with (10) or (11), states with $S \neq 0$. However, the average spin remains equal to zero. Indeed, let the exchange interaction be of the form

$$H = H_0 + H_1, \tag{12}$$

where $H_0 = I(\vec{S}_1\cdot\vec{S}_2)$ is the isotropic exchange and $I > 0$. We direct the Z axis along the vector $\vec{D}$. According to (5), S_Z commutes with $(A_ZS^2 - S^2A_Z)$ and accordingly with (12). Consequently, S_Z is a good quantum number and inasmuch as the operators S_x and S_y do not commute with the operator S_Z, we have $\bar{S}_x = \bar{S}_y = 0$. There remains the possibility of the appearance of $\bar{S}_Z$ along $\vec{D}$. But $S_Z = 0$ for the singlet state, and in order for S_Z to remain a good quantum number, H_1 can add to this state only states with $S_Z = 0$. Consequently, in the ground state that results from the singlet state, we have as before $S_Z = 0$. This explains the result recently obtained by direct calculation in [5] for the particular case $S_1 = S_2 = 1/2$.

The situation changes, however, if besides H_1 there acts in the system a factor stabilizing the antiparallel orientation of the spins. According to [3], this property of stabilizing "antiferromagnetic" ordering (in the classical sense) is possessed by the operator A_Z. We consider accordingly the Hamiltonian

$$H = I(S_1 \cdot S_2) + \frac{i}{4} D(AS^2 - S^2A) + pA_Z, \tag{13}$$

where p is the "molecular field." When $D = D_Z$, the operator S_Z commutes with H and in the ground state we have as before $\bar{S}_x = \bar{S}_y = \bar{S}_Z = 0$. If $D_\perp \neq 0$, however, a nonzero average spin becomes possible in the ground state, since S_Z does not commute, according to (5), with $(A_\perp S^2 - S^2 A_\perp)$. This can be verified also directly, by considering the wave function of the ground state of a system of two spins $S_1 = S_2 = 1/2$, which in the case of (13) has the following form

$$\Psi = F^{-1/2}\left[\left(\sqrt{\frac{1}{4}(I^2 + D^2) + p^2} + \frac{I}{2}\right)\Psi^s + \left(\frac{iD_Z}{2} - p\right)\Psi^T - \frac{i\bar{D}}{2\sqrt{2}}\Psi_\uparrow + \frac{i\bar{D}^+}{2\sqrt{2}}\Psi_\downarrow\right], \qquad (14)$$

where $F = (1/2)(I^2 + D^2) + 2p^2 + I\sqrt{(1/4)(I^2 + D^2) + p^2}$, Ψ^S is the singlet state, and $\Psi_\uparrow$, Ψ^T, and $\Psi_\downarrow$ are the triplet states with $S_Z = 1$, 0, and -1. In the state (14), the spin projection has the following mean values

$$\bar{S}_x = \frac{pD_y}{F}, \quad \bar{S}_y = -\frac{pD_x}{F}, \quad \bar{S}_Z = 0. \qquad (15)$$

Thus, under the joint action of pA_z and $\vec{D}[\vec{S}_1 \times \vec{S}_2]$ the mean value of the spin turns out to be different from zero, whereas each of them separately does not give rise to an average spin.

In conclusion we present an expression for the energy of the state (14):

$$E_1 = -\frac{I}{4} - \sqrt{\frac{1}{4}(I^2 + D^2) + p^2}, \qquad (16)$$

and also for the higher levels (for $I > 0$):

$$E_2 = E_3 = \frac{I}{4}, \quad E_4 = -\frac{I}{4} + \sqrt{\frac{1}{4}(I^2 + D^2) + p^2}. \qquad (17)$$

It is likewise easy to write out expressions for the corresponding wave functions.

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PHYSICAL SINGULARITY IN A SPACE FILLED WITH A NONIDEAL MEDIUM

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Submitted 22 June 1967
ZhETF Pis'ma 6, No. 6, 700-703 (15 September 1967)

We consider a space filled with a nonideal medium, i.e., a medium in which dissipative processes take place. The presence of such processes signifies that at each instant of time the medium is in a non-equilibrium state. We shall show that in this case the solution of the equations of gravitation, which depends on the maximum number of the physically arbitrary functions of three variables (p.a.f.), has a true singularity in the form of an anticollapse.

We write the energy-momentum tensor of the matter in the form [1]: