becomes possible in the ground state, since S_Z does not commute, according to (5), with $(A_\perp S^2 - S^2 A_\perp)$. This can be verified also directly, by considering the wave function of the ground state of a system of two spins $S_1 = S_2 = 1/2$, which in the case of (13) has the following form

$$\Psi = F^{-1/2}\left[\left(\sqrt{\frac{1}{4}(I^2 + D^2) + p^2} + \frac{I}{2}\right)\Psi^s + \left(\frac{iD_Z}{2} - p\right)\Psi^T - \frac{i\bar{D}}{2\sqrt{2}}\Psi_\uparrow + \frac{i\bar{D}^+}{2\sqrt{2}}\Psi_\downarrow\right], \tag{14}$$

where $F = (1/2)(I^2 + D^2) + 2p^2 + I\sqrt{(1/4)(I^2 + D^2) + p^2}$, Ψ^S is the singlet state, and $\Psi_\uparrow$, Ψ^T, and $\Psi_\downarrow$ are the triplet states with $S_Z = 1$, 0, and -1. In the state (14), the spin projection has the following mean values

$$\bar{S}_x = \frac{pD_y}{F}, \quad \bar{S}_y = -\frac{pD_x}{F}, \quad \bar{S}_Z = 0. \tag{15}$$

Thus, under the joint action of pA_z and $\vec{D}[\vec{S}_1 \times \vec{S}_2]$ the mean value of the spin turns out to be different from zero, whereas each of them separately does not give rise to an average spin.

In conclusion we present an expression for the energy of the state (14):

$$E_1 = -\frac{I}{4} - \sqrt{\frac{1}{4}(I^2 + D^2) + p^2}, \tag{16}$$

and also for the higher levels (for $I > 0$):

$$E_2 = E_3 = \frac{I}{4}, \quad E_4 = -\frac{I}{4} + \sqrt{\frac{1}{4}(I^2 + D^2) + p^2}. \tag{17}$$

It is likewise easy to write out expressions for the corresponding wave functions.

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PHYSICAL SINGULARITY IN A SPACE FILLED WITH A NONIDEAL MEDIUM

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We consider a space filled with a nonideal medium, i.e., a medium in which dissipative processes take place. The presence of such processes signifies that at each instant of time the medium is in a non-equilibrium state. We shall show that in this case the solution of the equations of gravitation, which depends on the maximum number of the physically arbitrary functions of three variables (p.a.f.), has a true singularity in the form of an anti-collapse.

We write the energy-momentum tensor of the matter in the form [1]:

$$T^{\mu\nu} = (\epsilon + p)\, u^{\mu} u^{\nu} + W^{\mu\nu}, \tag{1}$$

where

$$W^{\mu\nu} = \eta\,[u^{\mu};_{\sigma}(g^{\nu\sigma} - u^{\nu}u^{\sigma}) + u^{\nu};_{\sigma}(g^{\mu\sigma} - u^{\mu}u^{\sigma})] + (\zeta - \frac{2}{3}\eta)(g^{\mu\nu} - u^{\mu}u^{\nu})\, u^{\sigma};_{\sigma},$$

η and ζ are the coefficients of first and second viscosity, respectively. Usually one speaks of the viscosity of the medium in the case when the terms $W^{\mu\nu}$ are small in the co-moving coordinate system:

$$|W^{\mu\nu}| \ll |T^{\mu\nu} - W^{\mu\nu}| \,. \tag{2}$$

In the solution considered below, the condition (2) is not satisfied, and we shall therefore not use the term viscosity, and speak only of non-equilibrium processes.

We assume that the expansions of the components of the metric tensor have near the hypersurface $\tau \equiv x^0 - x^3 = 0$ the following form: *

$$-g_{ab} = a_{ab} + b_{ab}\tau + \ldots,\quad -g_{a3} = a_{a3}\tau^2 + b_{a3}\tau^3 + \ldots,\quad -g_{33} = a_{33}\tau^2 + b_{33}\tau^3 + \ldots \tag{3}$$

Assume that a physical singularity is reached at $\tau = 0$, and that the expansions for the energy density, the equilibrium pressure, and the velocity components are

$$\epsilon = \epsilon^{(-1)}\tau^{-1} + \epsilon^{(0)} + \ldots,\quad p = p^{(-1)}\tau^{-1} + p^{(0)} + \ldots,\quad u^{\sigma} = u^{\sigma(0)} + u^{\sigma(1)}\tau + \ldots \tag{4}$$

(all the coefficients in expansions (3) and (4) are arbitrary functions of the spatial coordinates).

The question is whether the assumptions (1), (3), and (4) are compatible with Einstein's equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -T^{\mu\nu}, \tag{5}$$

and if they are compatible, what is the number of the p.a.f. on which the corresponding solution depends. We shall show that the aforementioned assumptions are compatible with equations (5) under one additional condition: The singularity has the character of an anti-collapse. Mathematically this is equivalent to requiring that the quantity $u^{\sigma};_{\sigma}$, which is proportional to the rate of relative change of the proper volume, be positive on the hypersurface $\tau = 0$ and in its vicinity:

$$u^{\sigma};_{\sigma} \approx (u^{\sigma};_{\sigma})^{(-1)}\tau^{-1} > 0 \tag{6}$$

(on the hypersurface $\tau = 0$ itself, $u^{\sigma};_{\sigma}$ vanishes at $+\infty$). It will be shown that the corresponding solution depends on the maximum number of the p.a.f.

Allowance for the terms $W^{\mu\nu}$ in the tensor (1) increases the maximum number of the p.a.f. compared with the case of an ideal medium. If the coefficient η differs from zero, then the physical arbitrariness is determined by eleven functions of three coordinates. If $\eta = 0$ and $\zeta \neq 0$, then the number of p.a.f. is equal to nine. Without stopping to present a rigorous mathematical proof, we note that this result is understandable from physical considerations. Indeed, for an ideal medium the initial conditions should specify the distribution of the density of the matter, three velocity components, and four more quantities characterizing the free gravitational field [3,4]. A nonideal medium is described by the energy-momentum tensor (1), which includes also time derivatives of the velocity components. If $\eta \neq 0$, then the tensor (1) contains time derivatives of all four velocity components. By virtue of $u_\mu u^\mu = 1$, only three derivatives are independent, and it is for them that the initial data must be specified, thus increasing the physical arbitrariness by three functions. If $\eta = 0$ and $\zeta \neq 0$, then the tensor (1) contains one time derivative of the component u^0. In this case the number of p.a.f. increases by one.

In the first approximation in τ, we get from the equations in (5) with indices $\mu = a$, $\nu = b$; $\mu = 0$, $\nu = 3$; $\mu = a$, $\nu = 3$, respectively:

$$b_{ab} - a'_{ab} = 0, \quad u_o^{(1)} - u_o^{o'} = 0, \quad u^{a(1)} - u^{a(o)'} = 0 \tag{7}$$

(the prime denotes differentiation with respect to x^3). Taking (7) into consideration, we find that $G^{33} \sim \tau^{-2}$, $T^{33} \sim \tau^{-3}$, and the remaining $G^{\mu\nu} \sim \tau^{-1}$ and $T^{\mu\nu} \sim \tau^{-1}$. From (5) with the indices $\mu = \nu = 3$ we now obtain in the τ^{-3} approximation, after a number of transformations:

$$G^{33(-3)} \equiv 0 = a^{33}\left[p^{(-1)} - \left(\zeta + \frac{4}{3}\eta\right)(u^\sigma;_\sigma)^{(-1)}\right] \equiv T^{33(-3)}. \tag{8}$$

Since η and ζ are essentially positive, it follows from this equation that the quantities $p^{(-1)}$ and $(u^\sigma;\sigma)^{(-1)}$ have the same sign. Since $p \approx p^{(-1)}\tau^{-1} > 0$, we also have $u^\sigma;\sigma \approx (u^\sigma;\sigma)^{(-1)}\tau^{-1} > 0$ (see (6)). The remaining nine gravitation equations ($\mu \neq 3$, $\nu \neq 3$) relate in the τ^{-1} approximation the twenty functions that are contained therein (the coefficients of the series (3) and (4)), so that eleven functions remain arbitrary.** Since the solution does not admit of coordinate transformations connected with an arbitrary function of three variables, all eleven functions are p.a.f.

The condition (2) for $\mu = \nu = 3$ is of the form $(\zeta + 4/3\eta)(u^\sigma;\sigma)^{(-1)} \ll p^{(-1)}$. This relation does not contain the velocity-vector components, and consequently it will have the same form also in the co-moving coordinate system. Comparing this inequality with (8), we see that condition (2) is not satisfied.

It follows from (8) that the mixed component T_3^3 ($T_3^3 = T^{3\nu}g_{3\nu} = -a_{33}T^{33(-3)}\tau^{-1} + \ldots$) should be of the order of τ^0. Since the component T_3^3 does not contain in the τ^{-1} approximation any velocity-vector components, it represents a pressure (more accurately, a stress) along the x^3 axis. By virtue of the equations of gravitation, this pressure should remain finite as $\tau \to 0$. (This requirement is satisfied automatically for a dust-like medium, for which $p \equiv 0$. In the case of ideal matter, a finite pressure along the x^3 axis implies a finite pressure in other directions, since a medium which is in the equilibrium state has an isotropic pressure. Therefore the physical singularity in the metric (3) at $\tau = 0$ is possible in the former case, and furthermore with a complete set of p.a.f., but is impossible in the latter [5-7]. For a medium which is not in the equilibrium state, the indicated requirement is compatible with a singularity of the equilibrium pressure and of the energy density, by virtue of the fact that pressure anisotropy is possible in this case. In the solution under consideration, the matter is compressed at the initial instant of time into a "pancake" in the x^1, x^2 plane, and expansion begins along the x^3 axis, which is orthogonal near $\tau = 0$ to the x^1, x^2 plane. In the case of non-equilibrium expansion the pressure in the x^3 direction becomes smaller than the infinite equilibrium pressure by an infinite amount, and turns out to be a finite quantity. Thus, the physical meaning of the condition (6) is understandable: In the case of non-equilibrium compression along a certain axis, irreversible processes can only increase the pressure in this direction.

Certain properties of the solution considered above are similar to those possessed by the solution for weakly-interacting particles during initial stages of expansion in an anisotropically expanding world [8].

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* Expansions (3) were first proposed in [2]. The indices a and b run through the values 1 and 2.

** For more details see [5].

THEORY OF SKIN EFFECT IN AN ELECTRODE DISCHARGE

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The problem of skin flow of a high-frequency current is usually analyzed theoretically using as a model a half-space filled with plasma [1,2]. Such a model describes well the skin effect in an electrodeless discharge. However, as will be shown in the present communication, it is unsatisfactory in the case of an electrode discharge with sufficiently small distance between electrodes (which we shall denote by L). Namely, when $L \ll v_T/\omega$, where v_T is the thermal velocity of the electrons and ω is the frequency of the current, the thickness of the skin layer turns out to be much larger than the value calculated by the semi-bounded plasma model.

We investigate here the case when the thickness of the skin layer is small compared with the transverse dimension of the plasma column, and the plasma boundary can be regarded as plane. With respect to the electrodes it is assumed that they are parallel to each other