density, by virtue of the fact that pressure anisotropy is possible in this case. In the solution under consideration, the matter is compressed at the initial instant of time into a "pancake" in the x^1, x^2 plane, and expansion begins along the x^3 axis, which is orthogonal near $\tau = 0$ to the x^1, x^2 plane. In the case of non-equilibrium expansion the pressure in the x^3 direction becomes smaller than the infinite equilibrium pressure by an infinite amount, and turns out to be a finite quantity. Thus, the physical meaning of the condition (6) is understandable: In the case of non-equilibrium compression along a certain axis, irreversible processes can only increase the pressure in this direction.

Certain properties of the solution considered above are similar to those possessed by the solution for weakly-interacting particles during initial stages of expansion in an anisotropically expanding world [8].

The author is grateful to Ya. B. Zel'dovich and A. L. Zel'man for help in interpreting the obtained result, and also to A. G. Doroshkevich and I. D. Novikov for a discussion.

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* Expansions (3) were first proposed in [2]. The indices a and b run through the values 1 and 2.

** For more details see [5].

THEORY OF SKIN EFFECT IN AN ELECTRODE DISCHARGE

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Submitted 26 June 1967
ZhETF Pis'ma 6, No. 6, 704-707 (15 September 1967)

The problem of skin flow of a high-frequency current is usually analyzed theoretically using as a model a half-space filled with plasma [1,2]. Such a model describes well the skin effect in an electrodeless discharge. However, as will be shown in the present communication, it is unsatisfactory in the case of an electrode discharge with sufficiently small distance between electrodes (which we shall denote by L). Namely, when $L \ll v_T/\omega$, where v_T is the thermal velocity of the electrons and ω is the frequency of the current, the thickness of the skin layer turns out to be much larger than the value calculated by the semi-bounded plasma model.

We investigate here the case when the thickness of the skin layer is small compared with the transverse dimension of the plasma column, and the plasma boundary can be regarded as plane. With respect to the electrodes it is assumed that they are parallel to each other and perpendicular to the plasma boundary. The origin lies on the line of intersection of one of the electrodes with the plasma boundary, the x axis is perpendicular to this boundary, and the z axis is directed towards the second electrode. The system is infinite in the direction of the y axis. The external magnetic field is homogeneous and directed along the z axis.

We consider for simplicity the case when the magnetic field is so strong that the electron motion in the x and y directions can be neglected (it is easy to show that such an approximation is valid when the Larmor radius of the electrons is small compared with the thickness of the skin layer). The kinetic equation for the electrons then takes the form:

$$-i\omega f + v_z \frac{\partial f}{\partial z} - \frac{e}{m} E_z \frac{\partial f_0}{\partial v_z} = 0. \tag{1}$$

Here f is the perturbation of the distribution function, f_0 is the unperturbed distribution function, and the remaining symbols are standard.

The current is produced in the plasma under the influence of an external voltage source connected with the electrodes, the current being determined to a considerable degree by phenomena that occur on the electrode surfaces and in the Debye layers. For a qualitative description of these phenomena we use the boundary conditions

$$f(v_z)\,|_{z=0,L} = f(-v_z)\,|_{z=0,L} - g(v_z) E_z|_{z=0,L}, \tag{2}$$

where $g(v_z) > 0$ is a function that depends on the properties of the electrodes. Relation (2) shows that the current from the electrode (or to the electrode) appears only in the presence of an external electric field.

Putting $L \ll v_T/\omega$ and taking the symmetry of the problem into account, we get from (1) - (2):

$$j_z = \sigma E_z(x,0), \quad \frac{\partial \rho}{\partial z} = \frac{k_D^2}{4\pi} E_z(x, z), \tag{3}$$

where j_z and ρ denote respectively the current density and the charge density, and the quantities σ and k_D are defined by the relations:

$$\sigma = -e\int_0^\infty v_z g(v_z)\, dv_z, \quad k_D^2 = -\frac{8\pi e^2}{m}\int_0^\infty \frac{dv_z}{v_z}\frac{\partial f_0}{\partial v_z}.$$

The quantity σ can be called the "limiting conductivity." The solution (3) is valid if $\omega \ll \sigma \ll \omega_{pe} v_T/\omega L$. An investigation of the equations for the Debye layers shows that if the current is bounded by the space-charge effect (and not by the emissivity of the electrodes), then $\sigma \sim \omega_{pe}$.

Since our problem satisfies the condition for a quasistationary state, $\omega L/c \ll 1$, and the current j_z depends only on x, we can break up the electric field E into solenoidal and potential parts E_s and E_p:

$$E_s = (0, E_{sz}(x)), \; E_p = (E_{px}(x,z), E_{pz}(x,z)).$$

From Maxwell's equations it follows that

$$\frac{\partial^2 E_{pz}}{\partial x^2} + \frac{\partial^2 E_{pz}}{\partial z^2} = k_D^2 (E_{sz}(x) + E_{pz}(x,z)), \qquad (4)$$

$$\frac{d^2 E_{sz}}{dx^2} = -\frac{4\pi i \omega\sigma}{c^2}(E_{sz}(x) + E_{pz}(x,0)). \qquad (5)$$

E_p is due to the presence of space charge in the Debye layers near the electrodes and near the plasma-vacuum boundary. The charges become "aligned" in such a way that E_{pz} cancels E_{sz} almost completely within the volume of the plasma. With this, the field E_p in the layers themselves greatly exceeds E_s.

Integrating (4) with respect to z, we get

$$-2\frac{\partial E_{pz}}{\partial x}(x, 0) = k_D^2 L E_{sz}(x).$$

We have taken into account the fact that $E_{px} = 0$ on the surface of the conducting electrodes, and consequently $\int_0^L E_{pz} dz = 0$. Near the electrodes, Eq. (4) can be represented in the form

$$\frac{\partial^2 E_{pz}}{\partial z^2} = k_D^2 E_{pz}.$$

It follows therefore that

$$\frac{\partial E_{pz}}{\partial z}(x, 0) = -k_D E_{pz}(x,0) \text{ and } E_{pz}(x,0) = \frac{1}{2} k_D L E_{sz}(x).$$

We can now obtain from (5) an equation for the field distribution in the plasma:

$$\frac{d^2 E_{sz}}{dx^2} = -\frac{2\pi i \omega \sigma k_D L}{c^2} E_{sz}$$

We see that the effective skin-layer thickness δ is equal to $(c^2/\pi\sigma\omega k_D L)^{1/2}$. If we put $\sigma \sim \omega_{pe}$, then

$$\delta \sim \frac{c}{\omega_{pe}}\sqrt{\frac{v_T}{\omega L}}\ . \qquad (6)$$

Consequently, if the electron time of flight L/v_T is smaller than the period of the field, then the skin-layer thickness exceeds the value c/ω_{pe} obtained in the semibounded plasma approximation. This conclusion is of interest for many experiments aimed at the investigation of electrode discharges, for example [3].

Relation (6) admits of the following intuitive interpretation. It is well known that in a semibounded collision-dominated plasma (with effective collision frequency ν) the skin-layer thickness is of the order of $c\omega_{pe}(\nu/\omega)^{1/2}$. This yields formula (6) if ν is taken to be the frequency of the collisions of the electron with the electrodes.

In conclusion, the author is deeply grateful to L. I. Rudakov for numerous discussions that led to the formulation of the problem discussed above.

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ELASTIC π^+p SCATTERING THROUGH 90° ANGLE AND CONTRIBUTION OF Δ_δ REGGE TRAJECTORY IN THE s-CHANNEL

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Submitted 27 June 1967
ZhETF Pis'ma 6, No. 6, 707-712 (15 September 1967)

We develop in this paper the idea developed in [1], that resonances of the direct channel make a decisive contribution to the amplitude of elastic πp scattering through angles close to 90°. Attmepts to realize this idea in describing s-channel resonances by means of a sum of Breit-Wigner terms does not lead to the observed exponential decrease of the differential cross section for πp scattering through 90° with increasing energy, unless one resorts to additional assumptions which have no theoretical justification [1]. We used in this paper a different approach to this problem. The sum of the Breit-Wigner resonances was replaced by Regge's formula, which unifies in one term the contribution of the entire Regge trajectory on which the sequence of resonances lies.*

We consider elastic π^+p scattering through 90°. In this reaction, exchange is possible for resonances lying only on two Regge trajectories (Δ_δ: $I = 3/2$, $P = +$, $\sigma = -$; Δ_β: $I = 3/2$, $P = -$, $\sigma = +$). At the present time there is no confirmation of the recurrence of Regge poles for the Δ_β trajectory (only one resonance with mass 160 MeV is known). Consequently, in calculating the scattering amplitude

$$T(s, z) = f(s, z) + i\,\frac{\sigma\cdot[\mathbf{q}\times\mathbf{q}']}{q^2}\,\tilde{f}(s, z) \tag{1}$$

(q and q' are 3-momenta in the c.m.s. before and after scattering), we take into account only the contribution of the Δ_δ trajectory:

$$f(s, z) = \frac{\pi}{2q}\,\frac{\alpha(s)+1/2}{\cos\pi\alpha(s)}\,r_\alpha(s)\,[P_{\alpha(s)-1/2}(-z) - P_{\alpha(s)-1/2}(z)], \tag{2}$$

$$\tilde{f}(s, z) = -\frac{\pi}{2q}\,\frac{r_\alpha(s)}{\cos\pi\alpha(s)}\,[P'_{\alpha(s)-1/2}(-z) + P'_{\alpha(s)-1/2}(z)]. \tag{3}$$

Here $\alpha(s) = \alpha_1(s) + i\alpha_2(s)$ is the Regge Δ_δ trajectory, $r_\alpha(s)$ is its residue, and $z = \cos\theta$. The trajectory is described by the Chew-Frautschi line:

$$\alpha_1(s) = \alpha_1(0) + \alpha_1' s; \qquad \alpha_1(0) = 0.15, \quad \alpha_1' = 0.9\ \mathrm{GeV}^{-2} \tag{4}$$

Near the resonant value $s = M^2$ (M is the resonance mass), at which $\mathrm{Re}\,\alpha(M^2) = J$, the corres-