CALVANOMAGNETIC CHARACTERISTICS UNDER CONDITIONS OF THE NONLINEAR DE HAAS - VAN ALPHEN EFFECT

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At low temperatures, all quantities in a metallic single crystal are rapidly oscillating functions of the induction $\vec{B}$, and not of the external magnetic field $\vec{H}$. The small difference $\vec{B} - \vec{H} = 4\pi\vec{M}$ can become comparable with the oscillation period $\Delta\vec{B}$ and must be taken into account ($\vec{M}$ is the magnetization) [1-3]. It will be shown below that this causes an appreciable change in the form of the quantum oscillations of the resistance, leading even to the appearance of various anomalies. In addition, when diamagnetic domains appear in the sample [3], a change takes place in the character of the asymptotic behavior of the classical part of the electric-conductivity tensor, owing to the drift of the electrons in the inhomogeneous magnetic field near the domain boundaries (this is equivalent to the appearance of a layer of open trajectories). The accompanying resistance jumps can exceed in magnitude the Shubnikov oscillations and should be observed experimentally as an entirely new type of resistance oscillations.

We consider for simplicity a case when the contribution to the Shubnikov oscillations is made by one section. The form of the oscillations is determined by the equations

$$\sigma_{osc} = \sigma^{\circ}_{osc} \sin(2\pi \frac{B}{\Delta B} + \phi); \quad M = M_0 \sin 2\pi \frac{B}{\Delta B} . \tag{1}$$

Here φ is the phase shift between the oscillations of the electric conductivity and of the moment. If $\varphi = 0$, then the oscillations of $\sigma_{osc}(H)$ and $M(H)$ have the same form. If $\varphi \neq 0$ (or if several sections make contributions to the oscillations), then σ_{osc} and M are different functions of H, but the character of the singularities of $\sigma_{osc}(H)$ and $M(H)$ (discontinuities, points at which the derivative is anomalously large, regions of inhomogeneity) is the same, since it is connected only with singularities of B(H).

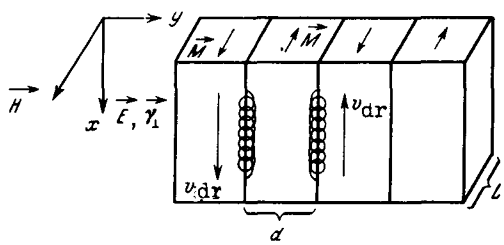

Fig. 1

If magnetic field domains exist in the samples, then the electrons crossing the domain walls drift transversely to the magnetic field (along the domain walls) (Fig. 1). The drift velocity is $v_{dr} \sim v_F R\nabla H/H$ [7], where v_F is the Fermi velocity and R the Larmor radius. Since the width of the transition region between domains is proportional to R, we have $v_{dr} \sim v_F 4\pi M/H$.

To estimate the additional electric conductivity $\Delta\sigma_{ik}$ connected with the drift, we use the well known formula for the electric conductivity in a magnetic field [4]

$$\sigma_{ik} = -e^2 \int \frac{\partial n_F}{\partial \epsilon} \, d\Gamma \frac{1}{\nu T^2} \int_0^T v_i(t) \int_{t-T}^{t} v_k(t-t') e^{-\nu t'} dt' \quad , \tag{2}$$

where $n_F(\epsilon)$ is the Fermi distribution, $d\Gamma$ the phase-volume element, T the Larmor radius, and ν the collision frequency. With the axes arranged as in Fig. 1 we have $\Delta\sigma_{xx} \sim \sigma_0(v_{dr}/v_F)^2$, where σ_0 is the electric conductivity in the absence of the magnetic field. To estimate the value $\overline{\Delta\sigma}_{xx}$ averaged over the sample it is necessary to multiply this expression by R/d, where d is the domain width, i.e., $\overline{\Delta\sigma}_{xx} \sim \sigma_0(4\pi M/H)^2 R/d$, with the remaining components $\overline{\Delta\sigma}_{ik} \approx 0$.

The asymptotic form of the transverse part of the electric conductivity tensor has in the absence of drift the form [4]

$$\begin{pmatrix} \gamma^2 a_{xx} & \gamma a_{xy} \\ -\gamma a_{xy} & \gamma^2 a_{yy} \end{pmatrix}, \quad \begin{pmatrix} \gamma^2 a_{xx} & \gamma^2 a_{xy} \\ \gamma^2 a_{yx} & \gamma^2 a_{yy} \end{pmatrix}, \quad \begin{pmatrix} \gamma^2 a_{xx} & \gamma a_{xy} \\ \gamma a_{yx} & a_{yy} \end{pmatrix}$$

a b c

$\gamma = R/l$, l is the mean free path, and $a_{ik} \sim \sigma_0$ $(\gamma \ll 1)$.

Cases a and b correspond to closed sections (in a the number of electrons is equal to the number of holes, in b it is not); in case c the section is open in the x-axis direction. In case a, the resistance increment has a maximum when the current is directed along the y axis: $(\overline{\Delta\rho}/\rho)_{max} \sim (4\pi M/H)^2 l^2/Rd$. (The increment is connected in this case with the change of the Hall field in the region between the domains.)

In cases b and c we have $\overline{\Delta\rho}/\rho \sim (4\pi M/H)^2 l^2/Rd$ for any current direction perpendicular to $\vec{B}$.

Inasmuch as the stratification into domains is periodic in 1/H, the resistance increment will have the form of jumps periodic in 1/H, with a period equal to that of the usual Shubnikov oscillations (Fig. 2). Let us compare the amplitude of these oscillations with the amplitude of the Shubnikov oscillations. For the Shubnikov oscillation amplitude we have [5] (under the condition $kT \lesssim 20\hbar\omega_H$) $\rho_{osc}/\rho \sim (\hbar\omega_H\epsilon_F)^{1/2} \sim (a/R)^{1/2}$, where ω_H is the Larmor frequency and a the interatomic distance. Recognizing that $4\pi M/H \sim (v_F/c)^2(\epsilon_F/\hbar\omega_H)^{1/2}$ (c is the speed of light), we get

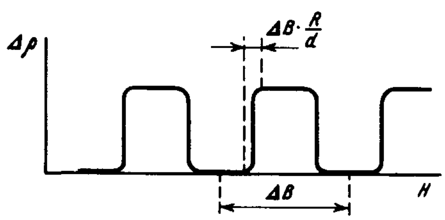

Fig. 2

$$\frac{\overline{\Delta\rho}}{\rho_{osc}} \sim \left(\frac{v_F}{c}\right)^4 \frac{l^2 R^{1/2}}{d\, a^{3/2}}. \quad (3)$$

There is at present no theory that yields the dependence of the domain dimensions on the sample thickness (Fig. 1). If we use the analogy with ferromagnets, then $d \sim \sqrt{L\,\Delta/\epsilon_0}$, where Δ is the surface tension on the boundaries between domains, and ϵ_0 is the additional energy density near the sample boundary.

Let us consider the simplest case: $\partial B/\partial H \sim 1$, $M_0 \sim \Delta B$. In this case $\Delta \sim RM_0^2$ [8]. Putting $\epsilon \sim M_0^2$, we get $d \sim \sqrt{RL}$. It then follows from (3) that

$$\frac{\overline{\Delta \rho}}{\rho_{osc}} \sim \left(\frac{v_F}{c}\right)^4 \frac{\ell^2}{L^{1/2} a^{3/2}} . \qquad (4)$$

Putting $v_F/c \sim 10^{-2}$, $a \sim 10^{-8}$ cm, and $L \sim 10^{-1}$ cm we find that when $\ell \gtrsim 10^{-2}$ cm the oscillations in question exceed the Shubnikov oscillations in amplitude.

Similar oscillations should occur if the periodic structure predicted by Azbel' [9] is observed.

An experimental observation of the oscillations described above would serve as a direct confirmation of the existence of domains or inhomogeneous structure, and would make it possible to determine their size.

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SCATTERING OF CONDUCTION ELECTRONS BY AN IMPURITY WITH SPIN

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As is well known, the problem of the scattering of a conduction electron by an impurity possessing a spin cannot be solved by perturbation theory. Solutions were obtained in [1,2] by the dispersion approach (both results agree near the Fermi surface), but they are not analytic with respect to the coupling constant.

We present here a solution of the problem for one particular case when the impurity spin is equal to unity; this solution is analytic in the coupling constant. When the coupling constant is positive, it agrees with the solution obtained in [2]. The method of obtaining the solution that follows, and also a generalization to the case of arbitrary impurity spin, will be published in a detailed article.

In [2] there were introduced the amplitudes $\alpha_\pm$, which in the case $\omega > 0$ are the scattering amplitudes in states with total angular momentum $J = S \pm 1/2$; the $\alpha_\pm$ are analytic functions of the energy. Unitarity conditions for these amplitudes were derived in [2]. We shall use in lieu of $\alpha_\pm$ the S-matrix elements $S_\pm = 1 + 2ik\alpha_\pm$, to which the following unitarity conditions apply: