$$\frac{\overline{\Delta\rho}}{\rho_{osc}} \sim \left(\frac{v_F}{c}\right)^4 \frac{\ell^2}{L^{1/2} a^{3/2}} . \quad (4)$$

Putting $v_F/c \sim 10^{-2}$, $a \sim 10^{-8}$ cm, and $L \sim 10^{-1}$ cm we find that when $\ell \gtrsim 10^{-2}$ cm the oscillations in question exceed the Shubnikov oscillations in amplitude.

Similar oscillations should occur if the periodic structure predicted by Azbel' [9] is observed.

An experimental observation of the oscillations described above would serve as a direct confirmation of the existence of domains or inhomogeneous structure, and would make it possible to determine their size.

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SCATTERING OF CONDUCTION ELECTRONS BY AN IMPURITY WITH SPIN

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As is well known, the problem of the scattering of a conduction electron by an impurity possessing a spin cannot be solved by perturbation theory. Solutions were obtained in [1,2] by the dispersion approach (both results agree near the Fermi surface), but they are not analytic with respect to the coupling constant.

We present here a solution of the problem for one particular case when the impurity spin is equal to unity; this solution is analytic in the coupling constant. When the coupling constant is positive, it agrees with the solution obtained in [2]. The method of obtaining the solution that follows, and also a generalization to the case of arbitrary impurity spin, will be published in a detailed article.

In [2] there were introduced the amplitudes $\alpha_\pm$, which in the case $\omega > 0$ are the scattering amplitudes in states with total angular momentum $J = S \pm 1/2$; the $\alpha_\pm$ are analytic functions of the energy. Unitarity conditions for these amplitudes were derived in [2]. We shall use in lieu of $\alpha_\pm$ the S-matrix elements $S_\pm = 1 + 2ik\alpha_\pm$, to which the following unitarity conditions apply:

$$|S_\pm|^2 = 1; \qquad \omega > 0$$

$$|S_\pm|^2 = \frac{(\mathrm{Re}\, u)^2 + (3 \mp 2)^2 k^2}{(\mathrm{Re}\, u)^2 + 9k^2}; \qquad \omega < 0 \tag{1}$$

$$\frac{S_+}{S_-} = \frac{u + 2ik}{u - 4ik}$$

k is the electron momentum; the function u is equal to [2]:

$$u = \frac{1 + a_+ a_- k^2}{b} - \frac{4p_0}{\pi} - \frac{2k}{\pi} \ln \left| \frac{k - p_0}{k + p_0} \right| + ik\epsilon(\omega), \tag{2}$$

$a_\pm$ are the Born scattering amplitudes introduced in [2], $b = (a_+ - a_-)/(2S + 1)$ is a parameter playing the role of the coupling constant in our problem, p_0 is the Fermi momentum, $\omega = E - E_F$, and $\epsilon(\omega)$ is the sign function. We shall consider only the case $p_0 a_\pm \ll 1$.

It is easy to verify that the unitarity conditions (1) are satisfied by the following functions

$$S_+ = \frac{u}{u - 2ik} D(\omega),$$

$$S_- = \frac{u}{u - 2ik} \frac{u - 4ik}{u + 2ik} D(\omega), \tag{3}$$

$$|D(\omega)|^2 = 1.$$

Expression (3) is similar in structure to the solution obtained in several papers for the Chew-Low equations [3,4].

It can be shown that near the Fermi surface

$$D = \frac{1 + ika}{1 - ika}; \qquad a = \frac{(S + 1)a_+ + Sa_-}{2S + 1}. \tag{4}$$

We see from (3) and (4) that the solution is analytic in b.

Let us compare our results with the solution obtained in [2]. We consider first the case $b > 0$. We introduce the function

$$f(\omega) = -\frac{i}{2k} \ln \frac{u}{u - 2ik}. \tag{5}$$

This function, like $u(\omega)$, has a cut in ω from $-E_F$ to ∞, and also cuts connected with the zeroes of the expression under the logarithm sign. We apply the Cauchy formula to $f(\omega)$. The contribution of the cut from $-E_F$ to ∞ is determined by the unitarity condition (1). When $b > 0$ the zeroes of $u(u - 2ik)^{-1}$ are on the physical sheet only when $\omega \ll -E_F$, and the contribution from these remote cuts can be easily taken into account in the same manner as in [2]. As a result we obtain for $S_\pm$

$$S_{\pm} = e^{2i\phi_{\pm}} \cdot \frac{1 + ika_{\pm}}{1 - ika_{\pm}} \; ; \quad b > 0 ,$$

$$\phi_{\pm} = -\frac{k}{4\pi} \int_{-E_F}^{0} \frac{d\omega'}{k'(\omega' - \omega)} \ln \frac{(\mathrm{Re}\, u)^2 + (3 \mp 2)^2 k'^2}{(\mathrm{Re}\, u)^2 + (3k')^2} . \tag{6}$$

When $b < 0$ additional zeroes of the function u appear on the physical sheet at $\omega = \pm i\epsilon_0$; $\epsilon_0 = E_F \exp(1/p_0 b)$, and an additional cut appears in $f(\omega)$. The contribution from this cut can be readily calculated, and we obtain

$$S_{\pm} = e^{2i\phi_{\pm}} \frac{\omega - i\epsilon_0}{\omega + i\epsilon_0} \frac{1 + ika_{\pm}}{1 - ika_{\pm}} \; ; \qquad b < 0 . \tag{7}$$

Expression (7) for $S_{\pm}$ is an analytic continuation of (6) in the coupling constant. Expression (6) was considered in [2] for both $b > 0$ and $b < 0$, and therefore the expression obtained there is not analytic in the coupling constant. Equation (7) can be rewritten in the form

$$S_{\pm} = e^{2i\phi_{\pm}} \frac{\dfrac{1}{a_{\pm}} + ik + \dfrac{\epsilon_0 k}{\omega - \dfrac{\epsilon_0}{ka_{\pm}}} \left[1 + \dfrac{1}{(ka \pm 1)^2}\right]}{\dfrac{1}{a_{\pm}} - ik + \dfrac{\epsilon_0 k}{\omega - \dfrac{\epsilon_0}{ka_{\pm}}} \left[1 + \dfrac{1}{(ka_{\pm})^2}\right]} . \tag{8}$$

Formula (8) coincides with the solution obtained in [2] if we introduce in the latter the Castillejo-Dalitz-Dyson pole. Thus, the requirement of analyticity in the coupling constant leads to appearance of a pole in the solution obtained in [2]. Let us consider the properties of the resultant solution. It can be shown that when $b > 0$ the scattering cross section decreases monotonically, on approaching the Fermi surface, from its usual value $4\pi(a^2 + 2b^2)$ far from the Fermi surface to $4\pi a^2$ at $\omega = 0$.

When $b < 0$ the section on the Fermi surface ($\omega = 0$) is also equal to $4\pi a^2$, but in this case it is no longer a monotonic function of the energy, and when $\omega \sim \epsilon_0$ its order of magnitude is $4\pi p_0^{-2} \gg 4\pi a^2$.

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