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The electric conductivity of a metal in a magnetic field is, as is well known, a superposition of a continuous component and an oscillating one. The oscillations of the conductivity in a magnetic field are called the Shubnikov - de Haas effect and were considered theoretically in [1]. The quantum oscillations obtained there for the conductivity in a magnetic field  $H_0$ , the observation of which calls for satisfaction of the conditions  $\mu H_0 \ll \epsilon_0$  and  $\pi^2 T < \mu H_0$ , are proportional to the quantity  $\partial \Delta M_z / \partial H_0$  ( $z \parallel H_0$ ) and the amplitude of the oscillations is of the order of  $\Delta \sigma_{ik}^{(qu)} \sim \Delta \sigma_{ik}^{(cl)} (\mu H_0 / \epsilon_0)^{1/2}$ , where  $\mu = e\hbar/2m^*c$  is the effective Bohr magneton and  $\epsilon_0$  is the Fermi energy. No account was taken there of the difference between the magnetic induction inside the metal and the magnetic field, although in fact all the kinetic and thermodynamic characteristics of a metal depend on the induction  $B$  [2]. For example, the classical part of the resistance of a metal in which the number of holes is equal to the number of electrons is  $\rho_{\alpha\beta} \sim B^2$  ( $\alpha, \beta = x, y; z \parallel H_0$ ) [3]. On the other hand, the induction, under appropriate boundary conditions, is an oscillating function of the external magnetic field. This leads to an additional oscillating term in the conductivity, connected directly with the influence of the de Haas - van Alphen effect on the resistance. Indeed, if the external magnetic field is parallel to the surface, then by virtue of the boundary conditions we have  $H_0 = H$  and the induction in the sample is  $B = H + 4\pi M(B) = H_0 + 4\pi M(B)$ . In this case the oscillating addition to the conductivity is proportional to  $\Delta M_z$ , and its amplitude is of the order of

$$\Delta \sigma_{ik}^{(H.A.)} \sim \Delta \sigma_{ik}^{(cl)} \frac{\Delta M_z}{H_0} \sim \Delta \sigma_{ik}^{(cl)} \left(\frac{v}{c}\right)^2 \left(\frac{\epsilon_0}{\mu H_0}\right)^{1/2}. \quad (1)$$

The oscillatory term indicated by us will dominate in the corresponding part of the conductivity if

$$\Delta \sigma_{ik}^{(H.A.)} / \Delta \sigma_{ik}^{(qu)} \sim \left(\frac{v}{c}\right)^2 \frac{\epsilon_0}{\mu H_0} \gtrsim 1. \quad (2)$$

This inequality is well satisfied in relatively weak fields, which in turn call for low temperatures ( $T \sim 0.1^\circ K$ ). For example, for a metal in which the number of holes is equal to the number of electrons, the additional oscillating term of the resistance is  $\Delta \rho_{\alpha\beta}^{(H.A.)} \sim 2B \Delta M_z$ . If the external field is perpendicular to the sample surface, then by virtue of the boundary conditions, the induction inside the metal is equal to the external magnetic field  $B = H_0$ , and the resistance oscillations coincide with those obtained in [1]. Thus, the considered effect of direct influence of the oscillations of the magnetic moment on the resistance depends on the direction of the field relative to the surface. The maximum amplitude of the indicated resistance oscillations is then attained in a parallel field.

If  $\chi = 4\pi(\partial M / \partial B) \sim (v/c)^2 (\epsilon_0 / \mu H_0)^{3/2} \gtrsim 1$ , then, as is well known, the homogeneously-

magnetized state becomes thermodynamically unstable [4]. At a certain  $H_0^{(i)}(T)$  ( $i = 1, \dots, N(T)$ ), two phases with different values of the induction,  $B_1(H_0^{(i)}, T)$  and  $B_2(H_0^{(i)}, T)$ , and with equal values of free energy, can coexist. Such a situation repeats in steps of  $\Delta H_0(H_0, T)$  which depend little on  $H_0$ . A first-order phase transition takes place (provided, of course, the surface energy of the interphase boundary is positive). With this, a domain structure can arise, determined entirely by the boundary conditions [4]. In the region  $(v/c)^2 \epsilon_0 / \mu H_0 > 1$  [region (1)] the oscillating part of the conductivity is proportional to  $\Delta M_z$ , and in the region  $(v/c) \epsilon_0 / \mu H_0 < 1$ ,  $\chi \geq 1$  [region (2)] it is proportional to  $\partial \Delta M_z / \partial H_0$ . If the external field is parallel to the surface of an infinite cylinder, no domain structure is produced. Therefore the conductivity is a periodic function of the field  $H_0$ , with discontinuities at the points  $H_0^{(i)}(T)$  in both region (1) and region (2) [4].

If the external field is perpendicular to the surface of an infinite plane-parallel plate, then domains are produced with walls parallel to the  $z$  axis and perpendicular to the  $y$  axis (see Fig. 1) with concentration  $C(B_1) = (B_2 - H_0)(B_2 - B_1)$  (the concentration is obtained from the boundary condition  $\bar{B} = H_0$ ). Let  $l \ll d$  and  $\delta \ll d$ , where  $l$  is the mean free path,  $\delta$  the width of the domain wall, and  $d$  the domain dimension. It is then

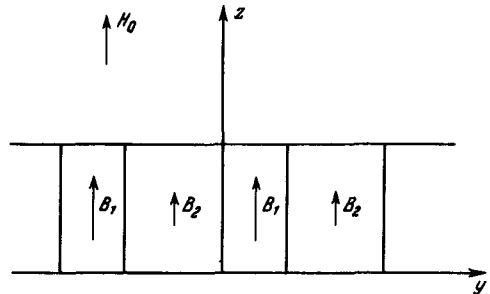


Fig. 1

meaningful to refer, in the case of the components  $\sigma_{xx}$  and  $\sigma_{yy}$ , to a quantity  $\bar{\sigma}_{ik}$  defined by

$$\bar{\sigma}_{ik} = C(B_1) \sigma_{ik}(B_1) + (1 - C(B_1)) \sigma_{ik}(B_2), \quad (3)$$

since we have in this approximation simply a parallel connection of "conductors" with  $\sigma_{ik}(B_1)$  and  $\sigma_{ik}(B_2)$ . For the same reason, it is convenient to introduce also the quantity  $\bar{\rho}_{yy}$ , defined by (3), since in this case we have a series connection of the "conductors." This leads to  $\bar{\sigma}_{xx}^{(cl)} = \sigma_{xx}^{(cl)}(H_0)$  and

$$\Delta \bar{\sigma}_{xx} = C(B_1) \Delta \sigma_{xx}(B_1) + (1 - C(B_1)) \Delta \sigma_{xx}(B_2),$$

i.e., the derivative with respect to the magnetic field is discontinuous at the points  $H_0^{(i)}(T)$  [4]. Similar formulas hold for the remaining components.

We also call attention to the fact that in the case of stratification into domains, the applied external electric field is redistributed inside the metal and becomes inhomogeneous. In our approximation we can easily find the indicated field distribution, namely,

$$E_y(y) = \begin{cases} E_y(B_1) & \text{for } B = B_1 \\ E_y(B_2) & \text{for } B = B_2 \end{cases},$$

where (by virtue of the continuity equation  $j_y = \text{const}$ ) we have for  $E(B_1)$  and  $E(B_2)$  the following system of equations:

$$\sigma_{yy}(B_1) E_y(B_1) = \sigma_{yy}(B_2) E_y(B_2); \quad C E_y(B_1) + (1 - C) E_y(B_2) = \bar{E}_y = L^{-1} \Delta \phi,$$

where  $L$  is the dimension of the sample and  $\Delta\phi$  is the potential difference at its ends. We note in conclusion that effects similar to those considered above take place also for the remaining kinetic coefficients of a metal in a magnetic field.

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#### PHYSICAL LIMITATIONS ON THE TOPOLOGY OF THE UNIVERSE

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In general relativity theory, rejection of the simple assumption of a flat Euclidian space raises naturally the question that the topology of space (three-dimensional as well as four-dimensional space-time) can differ from the simple topology of flat space for an open world or the topology of a sphere for a closed world.

Shortly following Einstein's first paper [1] on the cosmological problem, in which he constructed a static cosmological model with a closed spherical three-dimensional space, Klein [2] indicated that a three-dimensional space with the same metric can also be elliptic,\* i.e., it can have on the whole other properties than a spherical space.

The question of the connectivity of the space as a whole and of its topology is continuously mentioned in the literature (see [3]).

The nonstationary nature of the universe and the probable existence of a singularity in the past obviously limits the region accessible to observation and hinders a direct observational investigation of the topology, say by observing the same remote object in opposite directions. Interest attaches therefore to those limitations that can be imposed on the topology by starting from considerations that do not depend on astronomic observations.

One such limitation is the requirement that causality be satisfied. This requirement is incompatible with manifolds containing closed timelike world lines (see [4,5]).

The purpose of the present note is to emphasize that recent discoveries in the physics of elementary particles, which make possible absolute definitions of "right" and "left," show that a real physical three-dimensional space cannot be non-orientable (this cannot be refuted a priori). It is known that non-orientable three-dimensional spaces are contained, for example, among the 18 possible spaces of constant zero curvature (among both the open and the closed ones)\*\* [6].

Suveges [6] emphasizes that in a non-orientable space inversion is a continuous transformation (and not discrete, as in an orientable space).