ties of space to physics was emphasized by Wheeler a number of times [10]. In this note we speak of non-orientability on a cosmic scale.

**** If the mirror-particle hypothesis turns out to be valid (see [8]), then symmetry between right and left will be restored. In this case a non-orientable space becomes possible under the condition that a circuit over a non-orientable contour corresponds to CPA inversion (A - transformation into a mirror particle).

UPPER LIMIT OF ELEMENTARY LENGTH

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1. Lindenbaum [1] recently derived from a comparison of new data on forward πp scattering and the dispersion relations the following estimate

$$l < 0.7 \times 10^{-15} \text{ cm} \qquad (1)$$

Here l is the elementary length, i.e., a quantity determining the dimensions of the space-time region in which violation of the general principles of elementary-particle theory is possible.

Lindenbaum's result is of considerable interest, being more effective by approximately one order of magnitude than the earlier estimates based on verification of quantum electrodynamics (cf., e.g., [2]). However, the relation $l < 1/\omega_L$ (ω_L is the energy in the laboratory system) used in [1] and leading to (1) is subject to serious objections and can hardly be justified.

2. The correct method for determining the upper limit of l, the one usually employed in the reduction of the results of electrodynamic experiments, is based on introducing into the expression for the measured quantity (A) certain factors that violate the general principles of the theory, most frequently locality. As a result, A acquires a correction term δA, which depends on l. If there is no discrepancy between experiment and local theory (within an accuracy Δ), then we have the obvious inequality

$$|\delta A(l)| < \Delta, \qquad (2)$$

which can be used to determine the upper limit of l.

Unfortunately, the use of this method in our problem is made very difficult by the impossibility of exactly calculating $|\delta A|$ (strong interactions). It is possible, however, to obtain for this quantity a lower bound, from which definite conclusions can be drawn regarding the plausibility of the inequality (1).

3. To obtain the nonlocal correction to the dispersion relations, we shall use the nonlocal scheme proposed by Leznov and the author [3] and satisfying the requirements of relativism, correspondence, convergence, and (with certain stipulations) microcausality (see also [4]).* This scheme is based on an expanded Hilbert space of states, containing besides the usual states also auxiliary unphysical states with negative norm and with masses $\kappa \geq 1/l$. We construct the local theory on this expanded space in the usual manner, and use in lieu of the usual field operators φ, ψ, etc. the combinations

$$\phi - \sum_\kappa \tilde{\phi}_\kappa C_\kappa , \qquad \psi - \sum_\kappa \tilde{\psi}_\kappa C'_\kappa \quad , \tag{3}$$

where $\tilde{\phi}$ and $\tilde{\psi}$ are unphysical operators, and $\sum_\kappa C_\kappa^2 = \sum_\kappa C_\kappa'^2 = 1$.

The corresponding scattering matrix is local and unitary on the expanded space, and therefore the forward πp scattering amplitude obeys the usual dispersion relation, the only difference being that the total cross section in the absorptive part includes all the transitions, including the unphysical ones, $\sigma_t = \sigma - \tilde{\sigma}$, where σ is the usual cross section and $\tilde{\sigma}$ is the cross section of those processes in which there is at least one unphysical particle in the final state (the minus sign is connected with the negative sign of the norm).

The transition to the nonlocal theory in physical space is effected simply by so changing the imaginary part of the scattering matrix element, as to make it unitary on the physical space. It is clear therefore that the dispersion relations for the real part of the amplitude differ from the usual ones only in that σ is replaced by σ_t. Accordingly, the correction of interest to us is of the form **

$$|\delta A| = \frac{1}{2} |\delta (D_+ + D_-)| = \frac{k^2}{4\pi^2} \left| \int_0^\infty \frac{dk'}{k'^2 - k^2} (\tilde{\sigma}_+ + \tilde{\sigma}_-) \right| , \tag{4}$$

where $D_\pm$ is the real part of the amplitude of forward $\pi^\pm p$ scattering.

4. We confine ourselves for simplicity to the case of one unphysical fermion ($C'_\kappa = \delta_{\kappa,1/\ell}$). Since integration in (4) begins with the value $1/2M\ell^2$ (M is the proton mass), we see, assuming that $2Mk\ell^2 < 1$ (this will be confirmed by the result), that the integrand in (4) has a constant sign. We therefore only strengthen the inequality (2) by leaving in $\tilde{\sigma}_+ + \sigma_-$ only the cross section σ_0 for the simplest process

$$\pi^- + p \rightarrow \text{unphysical fermion} \tag{5}$$

and omitting k^2 from the denominator. As a result we have

$$\frac{k^2}{4\pi^2} \int \frac{dk'}{k^2} \tilde{\sigma}_0 \ (k') < \Delta \ .$$

Introducing the renormalized coupling constant for the process (5) (according to (3), it coincides in the case of one unphysical particle with the usual meson-nucleon coupling constant), we get as a result of simple manipulations

$$\tilde{\sigma}_0 = \frac{\pi g^2}{4M} \delta (k - 1/2M\ell^2).$$

From this we get

$$\ell < \left(\frac{4\pi\Delta}{g^2 k^2 M}\right)^{1/4} = \left(\frac{16\pi\Delta M}{g^2}\right)^{1/4} 1/\omega_c \ , \tag{6}$$

where ω_c is the c.m.s. energy. Putting $g^2 \simeq 15$ and taking from the plots of [1] the estimate $\Delta \sim 0.1/M$, we get

$$l < 2 \times 10^{-15} \text{ cm}. \qquad (7)$$

Inclusion of the remaining unaccounted-for processes can lower the upper limit of l. There is therefore every reason for assuming that the estimate (1), although obtained from an unproved formula, is nonetheless apparently not very far from the truth.

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* Another nonlocal scheme was used by Blokhintsev and Kolerov [5] to solve a similar problem.

** The dispersion relation for $\frac{1}{2}(D_- - D_+)$ leads to similar results.

AMPLITUDE SINGULARITIES IN TWO-PARTICLE NUCLEAR REACTIONS

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In a number of experimental papers (see [1,2]) the energy dependence of the differential cross sections of the reactions

$$\begin{array}{ll} T\,(p,p)\,T & T\,(p,n)\,He^3 \\ He^3\,(n,p)\,T & He^3\,(n,n)\,He^3 \\ T\,(p,\gamma)\,He^4 & He^4\,(\gamma,p)\,T \end{array} \qquad (1)$$

reveals extrema that are interpreted as extrema corresponding to excited states of the compound nucleus He^4. These extrema have widths on the order of 1 MeV and lie in the excitation-energy region 20 - 30 MeV. We show in this paper that the Feynman triangular diagram shown in Fig. 1 yields a number of extrema in the differential cross section of the reaction $A + x \to B + y$. Their widths are of the order of 2 - 5 MeV, and their position does not depend on the scattering angle. The height (depth) of the extrema depends strongly on the presence in the reaction amplitude of terms corresponding to other diagrams, that depend on the emission angle θ of the reaction products and vary slowly with the initial energy. In the case of the reactions (1) the extrema determined by the diagram 1 lie at initial energies (in the reaction c.m.s.) that coincide with the position of the hypothetical levels of He^4.

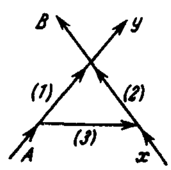

Fig. 1. Triangular diagram $T_\Delta(\omega)$.

As shown in [3], the triangular diagram of Fig. 1 is best applied using the variables ξ and λ, defined by the formulas

$$\xi = \frac{m_2}{m_3} \frac{m_A}{m_B + m_y} \frac{\omega - Q}{\epsilon} \qquad (2)$$

$$\lambda = \frac{m_1}{m_3} \frac{m_x(\omega - Q_0)}{(m_B + m_y)\,\epsilon},$$