$$l < 2 \times 10^{-15} \text{ cm.} \qquad (7)$$

Inclusion of the remaining unaccounted-for processes can lower the upper limit of l. There is therefore every reason for assuming that the estimate (1), although obtained from an unproved formula, is nonetheless apparently not very far from the truth.

I am grateful to A. A. Komar, A. N. Leznov, and V. Ya. Fainberg for useful discussions.

[1] S. Lindenbaum, BNL Preprint 11175, Coral Gables, 1967.
[2] R. Gatto, Preprint, TH 65/22, Florence, 1966.
[3] A. N. Leznov and D. A. Kirzhnits, Zh. Eksp. Teor. Fiz. 48, 622 (1965) [Sov. Phys.-JETP 21, 411 (1965)].
[4] D. A. Kirzhnits, Usp. Fiz. Nauk 90, 129 (1966) [Sov. Phys.-Usp. 9, 692 (1967)].
[5] D. I. Blokhintsev and G. I. Kolerov, Nuovo Cimento 34, 163 (1964).

* Another nonlocal scheme was used by Blokhintsev and Kolerov [5] to solve a similar problem.

** The dispersion relation for $\frac{1}{2}(D_- - D_+)$ leads to similar results.

AMPLITUDE SINGULARITIES IN TWO-PARTICLE NUCLEAR REACTIONS

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In a number of experimental papers (see [1,2]) the energy dependence of the differential cross sections of the reactions

$$\begin{array}{ll} T\,(p,p)\,T & T\,(p,n)\,He^3 \\ He^3\,(n,p)\,T & He^3\,(n,n)\,He^3 \\ T\,(p,\gamma)\,He^4 & He^4\,(\gamma,p)\,T \end{array} \qquad (1)$$

reveals extrema that are interpreted as extrema corresponding to excited states of the compound nucleus He^4. These extrema have widths on the order of 1 MeV and lie in the excitation-energy region 20 - 30 MeV. We show in this paper that the Feynman triangular diagram shown in Fig. 1 yields a number of extrema in the differential cross section of the reaction $A + x \to B + y$. Their widths are of the order of 2 - 5 MeV, and their position does not depend on the scattering angle. The height (depth) of the extrema depends strongly on the presence in the reaction amplitude of terms corresponding to other diagrams, that depend on the emission angle θ of the reaction products and vary slowly with the initial energy. In the case of the reactions (1) the extrema determined by the diagram 1 lie at initial energies (in the reaction c.m.s.) that coincide with the position of the hypothetical levels of He^4.

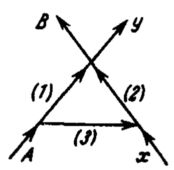

Fig. 1. Triangular diagram $T_\Delta(\omega)$.

As shown in [3], the triangular diagram of Fig. 1 is best applied using the variables ξ and λ, defined by the formulas

$$\xi = \frac{m_2}{m_3}\,\frac{m_A}{m_B + m_y}\,\frac{\omega - Q}{\epsilon} \qquad (2)$$

$$\lambda = \frac{m_1}{m_3}\,\frac{m_x(\omega - Q_o)}{(m_B + m_y)\,\epsilon},$$

where m_i are the masses of the particles i, ω is the energy of the particles B and y in their c.m.s., and

$$\begin{aligned} \epsilon &= m_1 + m_3 - m_A \\ Q &= m_A + m_x - m_B - m_y \\ Q_0 &= m_A + m_x - m_B - m_y \, . \end{aligned} \tag{3}$$

The amplitude of the reaction M is the sum of a part $M_\Delta(\omega)$, which varies rapidly as a function of ω and corresponds to the diagram of Fig. 1, and a term $M_R(\theta, \omega)$, which is generated by other diagrams. We have:

$$|M|^2 = |M_R|^2 + |M_\Delta(\omega)|^2 + 2 Re M_\Delta(\omega) \; Re M_R(\theta,\omega) + 2 Im M_\Delta(\omega) \; Im M_R(\theta,\omega), \tag{4}$$

where, according to [3,4]:

$$M_\Delta(\omega) = C f_\Delta(\xi,\lambda) \equiv C f_\Delta(\omega)$$

$$f_\Delta(\xi,\lambda) = \begin{cases} \dfrac{1}{\sqrt{\lambda}} \left[\dfrac{1}{2} \ln \dfrac{1 + (\sqrt{\xi} - \sqrt{\lambda})^2}{1 + (\sqrt{\xi} + \sqrt{\lambda})^2} + i \; \mathrm{arctg} \; \dfrac{2\sqrt{\lambda}}{1 + \xi - \lambda} \right] & \xi \geqslant 0 \\ \dfrac{2}{\sqrt{\lambda}} \; \mathrm{arctg} \; \dfrac{\sqrt{\lambda}}{1 + \sqrt{-\xi}} & \xi \leqslant 0. \end{cases}$$

The constant C was calculated in [4]. The argument ω in the function $M_R(\theta, \omega)$ can be replaced by $\omega = \omega_e$ in the vicinity of the extremum ω_e of the amplitude $M_\Delta(\omega)$.

The quantities Im $f_\Delta |f_\Delta|^2$ as functions of ω have a peak at $\xi = 0$ (extremum I (EI) with width of several MeV, corresponding to a root singularity of the function $f_\Delta(\omega)$). The quantity Re $f_\Delta(\omega)$ has an extremum II (EII), the approximate position of which is given by

$$\xi \approx 1 + \lambda. \tag{5}$$

In reactions (1), $\lambda \ll 1$. The exact position of EII is determined by tabulating $f_\Delta(\xi, \lambda)$ (see the level schemes of Re $f_\Delta(\xi, \lambda)$ and Im $f_\Delta(\xi, \lambda)$ in [3,4]). The forms of EI and EII are given in [4]. We see from (4) that either EI or EII will appear, depending on the relation between Re $M_R(\theta, \omega_e)$ and Im $M_R(\theta, \omega_e)$. The forms of EI and EII, their height, and their position in the differential cross section yield information on the relative magnitude and signs of Im $M_R(\theta, \omega_e)$ and Re $M_R(\theta, \omega_e)$. For example, in the case when sign Re $M_R(\theta, \omega_{EI})$ = -sign Re $M_\Delta(\omega_{EI})$ and sign Im $M_R(\theta, \omega_{EI})$ = sign Im $M_\Delta(\omega_{EI})$ the maximum EI will have a slightly smaller width than that predicted by the level schemes of Re $f_\Delta(\xi, \lambda)$ and Im $f_\Delta(\xi, \lambda)$.

The table lists the positions of EI and EII for the reactions (1). The energy is reckoned from the ground state of He^4.

The diagrams $M_\Delta(\omega)$ corresponding to the reaction $He^4 + \gamma \rightarrow p + T$ are obtained from the diagrams given in the table for the reaction $p + T \rightarrow He^4 + \gamma$ by reversing the direction of

Table of extrema I and II corresponding to different diagrams of type $T_\Delta(\omega)$ contributing to the amplitude of the reactions (1)

Reaction	A	x	B	y	1	2	3	EI MeV	EII MeV
$p + T \to p + T$	T	P	T	P	n	He^3	d	20.5	27
$p + T \to n + He^3$			(He^3)	(n)	d	d	n	23.8	30
$n + He^3 \to n + He^3$	He^3	n	He^3	n	p	T	d	–	25.3
$n + He^3 \to p + T$			(T)	(p)	d	d	p	23.8	29
$p + T \to \gamma + He^4$	T	p	He^4	γ	n	He^3	d	20.6	26
$\gamma + He^4 \to p + T$					d	d	n	23.8	30
$p + He^3 \to p + He^3$	He^3	p	He^3	p	p	He^3	d	–	24.7

motion of all the particles. The analytic form of these Feynman diagrams and the positions of EI and EII coincide in this case for both reactions. In Fig. 2 are gathered the theoretical results on the positions of EI and EII, obtained in this paper (solid lines) and the data on the positions of the experimentally observed extrema, heretofore interpreted as resonances, corresponding to the levels of the compound nucleus He^4 (see [1,2]). As seen from Fig. 2, at energies higher than 24 MeV the extremum positions predicted in this paper agree with the hypothetical "He^4 levels." The question of the physical nature of the irregularities in the cross sections of the reactions (1) cannot be regarded as resolved at present. Its solution calls for more exact experimental data on the differential cross section as a function of the kinematic variables.

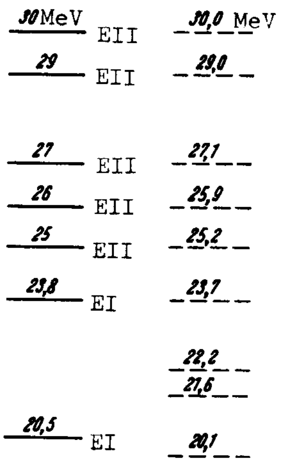

Fig. 2. Scheme comparing the positions of the extrema of the diagram $T_\Delta(\omega)$ (solid lines) and of the experimentally known extrema in the cross sections of the reactions (1) (dashed lines)

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[1] I. Ya. Barit, Paper at Problem Symposium on Nuclear Physics, Tbilisi, April 1967.
[2] P. E. Argan, G. C. Mantovani, P. Marazzini, et al., Suppl. Nuovo Cimento 8, 245 (1965).
[3] E. I. Dubovoi and I. S. Shapiro, Zh. Eksp. Teor. Fiz. 53, 1395 (1967) [Sov. Phys.-JETP 26 (1968), in press].
[4] E. I. Dubovoi and I. S. Shapiro, Zh. Eksp. Teor. Fiz. 51, 1251 (1966) [Sov. Phys.-JETP 24, 839 (1967)].