FREQUENCY SPECTRUM OF SELF-FOCUSING LIGHT PULSES

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The field of the powerful light filaments produced as a result of self-focusing [1-3] is capable of changing the refractive index of the medium by a noticeable amount ($\Delta n/n_0 \sim 10^{-2}$ - 10^{-1} and more). It will be shown in this note that such a nonlinearity can lead to a broadening of the frequency spectrum of the short (10^{-8} - 10^{-9} sec) optical pulses used in self-focusing experiments by a large number of times, without noticeably distorting the pulse envelope amplitude.

The propagation of time-modulated waves, particularly pulses, in a nonlinear dielectric was considered from different points of view in a number of papers [4 - 10]. For the initial calculations it is sufficient to use simple first-approximation formulas describing the propagation of a quasiharmonic plane wave [7]. Namely, representing the field of the wave in the form:

$$E = A \exp[i(\omega t - kz + \phi] + K_{conj},$$

where $A(z, t)$ and $\phi(z, t)$ are the slowly varying amplitude and phase, we can obtain

$$A = \tilde{A}(\xi), \quad \phi = -\frac{n'(\xi)}{n_0} kz + \tilde{\phi}(\xi). \tag{1}$$

Here $\xi = t - z/v$, v is the group velocity, $n = n_0 + n'$ is the refractive index, and $n'(A)$ is its nonlinear part; the functions $\tilde{A}$ and $\tilde{\phi}$ are prescribed by the parameters of the wave entering the medium at $z = 0$. Since n' depends on the time ($n' \sim A^2$ when the nonlinearity is small, but formula (1) holds true even when saturation effects are taken into account), it follows that the wave is phase (frequency) modulated during the course of its propagation even if $\tilde{\phi} \equiv 0$.

Let us estimate the change in the width and form of the pulse spectrum in accordance with (1). We assume first that $\tilde{\phi} \equiv 0$, and then the width Δ of the spectrum of the incident pulse of duration T is of the order of T^{-1}. If, however, the maximum phase deviation in time ϕ_{max} greatly exceeds π, then, as is well known, the width of the pulse spectrum is close not to T^{-1}, but to the maximum frequency deviation $\omega' = \partial\phi/\partial t$ (we note that when $\phi_{max} >> \pi$ the "modulation index" $m \sim \omega'_{max}T$ is large). The "critical" distance z_1 at which ϕ_{max} becomes of the same order as π, is equal to n_0/kn'_{max} according to (1).

It follows from the foregoing that

$$\Delta \simeq T^{-1} (z \leqslant z_1), \quad < \Delta \simeq \frac{kz}{n_0} \left(\frac{\partial n'}{\partial t}\right)_{max} \quad (z \gtrsim z_1). \tag{2}$$

In an unfocused beam ($n' \sim 10^{-5}$ - 10^{-6}), z_1 is of the order of 10 - 100 cm, and under ordinary experimental conditions (length of cell with nonlinear liquid not exceeding 20 - 50 cm) the spectrum broadening should not be large, although the emission line shape may be noticeably altered by the effect under consideration. On the other hand, for a focused channel ($n' \sim 10^{-2}$ - 10^{-1}) we get $z_1 \simeq 10^{-2}$ - 10^{-3} cm, and the spectrum of the emerging radiation should be much broader than the spectrum of the incident radiation. Thus, for $\lambda = 1\ \mu$, $T = 10^{-8}$ sec, and $n'_{max} = 10^{-1}$ we get already at a distance z = 2 cm the value $\Delta \approx 2 \times 10^{12}$ rad/sec and $(\Delta/\omega) \approx 10^{-3}$, whereas in the incident pulse $(\Delta/\omega) \approx 5 \times 10^{-7}$.

If $\tilde{\phi} \neq 0$ (for example, as a result of intermode beats in the laser), then all the above remains in force; it is only necessary to take into account the fact that when $\tilde{\phi}_{max} > \pi$ the width of the spectrum can exceed T^{-1} from the very beginning.

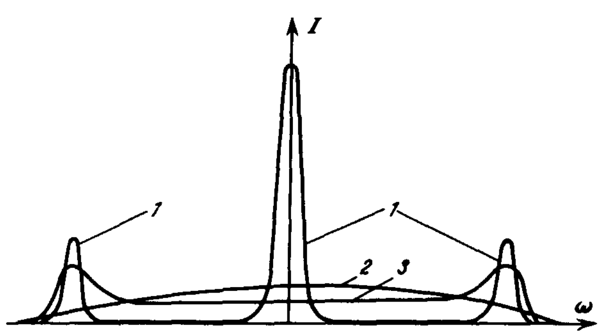
The shape of the spectral line $I(\omega)$ of a radiation pulse passing through a nonlinear medium depends on the time variation of the intensity $I(t)$ (the shape of the pulse). In the case of interest to us, $m \sim \omega'_{max}T \gg 1$ (relatively deep and slow changes in the frequency), we can obtain approximately the distribution of the power over the spectrum in the following fashion. We break up the function $\omega'(t)$ into small sections δt in which the frequency change $\delta\omega$ is such that $(\delta\omega\delta t) \gg 1$, and hence it can be assumed that the energy of these sections is equal to the energy of the corresponding spectral interval of width $\delta\omega$ ("running spectrum"). Assuming that $I(\omega)\delta\omega = I(t)\delta t$, we get $I(\omega)$ in parametric form*

$$I(\omega) = \Sigma \frac{I(t)}{\dot{\omega}(t)}, \quad \omega = \omega(t) , \tag{3}$$

where the sum is taken over different sections with identical ω (such an ambiguity arises only if the variation of $\omega(t)$ is nonmonotonic). It can be shown that the condition for the applicability of (3) is of the form $|\ddot{\omega}| \ll |\dot{\omega}|^{3/2}$, i.e., Eq. (3) is no longer valid near the extrema of the function $\omega(t)$; to estimate the order of magnitude of $I(\omega)$ in the vicinity of the extremal point, it is necessary to substitute in (3) the "limiting" value of t for which $|\ddot{\omega}| \approx |\dot{\omega}|^{3/2}$.

Depending on the form of the function $n'(\xi)$ (which is approximately proportional to the intensity of the wave field), the line shape $I(\omega)$, described by formulas (1) and (3), can vary greatly, viz., either the pulse energy is distributed relatively uniformly over the spectrum, or else there are sharply pronounced maxima near the individual frequencies for which $\dot{\omega} = 0$ (see the figure). If the shape of the pulse entering the medium is asymmetrical in time, then the spectral line of the transmitted pulse will also be asymmetrical.

Formula (1), which does not take into account the deformation of the amplitude wave, is valid until ω' reaches a value on the order of $\omega\sqrt{n'_{max}/n_0\kappa}$, where $\kappa = (\omega/v)(dv/d\omega)$ is the dispersion parameter [7]. According to (1), this occurs at a distance $z_2 \approx v\sqrt{n'_{max}/\kappa} \times (n'_{max})^{-1}$. The remainder of the process depends on the sign of κ; when $(n'\kappa) > 0$, self-modulation of the light is possible [7], and when $(n'\kappa) < 0$ there can be produced abrupt drops in the intensity inside the pulse, in



Approximate form of spectral lines of pulses passing through a nonlinear medium (m >> 1). Curves 1, 2, and 3 correspond to trapezoidal, parabolic, and Gaussian intensity profiles at equal energy and characteristic pulse length.

the form of envelope shock waves [4,5]. We note that in the range $\lambda \sim 0.5 - 1\ \mu$ we have for the customarily employed liquids $\kappa < 0$; thus $\kappa \simeq -0.07$ and $\kappa = -0.14$ for benzene and carbon disulfide, respectively, at $\lambda \sim 0.7\ \mu$ and a temperature 18 - 20°C. For the pulse parameters indicated above we have $z_2 \gtrsim 100$ cm, and (1) is applicable practically at all times, but if some fast processes are present in the initial pulse (including interaction with Raman-scattering components) it is necessary to take into account the variation of the amplitude envelope. We note that the change in the wave spectrum can be strongly asymmetrical here.

The foregoing has pertained to a plane wave, but for a channel with fixed (quasistationary) transverse field distribution we can draw similar conclusions. Thus, using the results of [9, 11] for a symmetrical three-dimensional channel and neglecting the relaxation time of the medium, we can easily obtain formula (1), where $n'(\xi)$ is equal to half the value of n' on the channel axis.

It is possible that the mechanism considered here explains to some degree the strong broadening of the spectrum of optical radiation in self-focusing, which was observed in some experiments [12]. We note also that the presence of a reactive nonlinearity in the resonator [8] (for example, due to the presence of nonlinear filters), can lead to a noticeable (albeit weaker than in self-focusing) broadening of the laser emission spectrum, especially for single-mode lasers.

In conclusion, I am grateful to A. V. Gaponov for discussion of the problems considered here.

[1] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. 42, 1567 (1962) [Sov. Phys.-JETP 15, 1088 (1962)].
[2] R. Chiao, E. Garmire, and C. Townes, Phys. Rev. Lett. 13, 479 (1964).
[3] V. I. Talanov, Izv. VUZov, Radiofizika, 7, 564 (1964).
[4] L. A. Ostrovskii, Zh. Tekhn. Fiz. 33, 905 (1963) [Sov. Phys.-Techn.Phys. 8, 679 (1964)].
[5] L. A. Ostrovskii, URSI Symposium, Delft, Netherlands, 1965.
[6] Ya. B. Zel'dovich and Yu. P. Raizer, ZhETF Pis. Red. 3, 137 (1966) [JETP Lett. 3, 86 (1966)].
[7] L. A. Ostrovskii, Zh. Eksp. Teor. Fiz. 51, 1189 (1966) [Sov. Phys.-JETP 24, 797 (1967)].
[8] L. A. Ostrovskii, Paper at All-union Symposium on Nonlinear Optics, Novosibirsk, 1966.
[9] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Zh. Eksp. Teor. Fiz. 51, 296 (1966) [Sov. Phys.-JETP 24, 198 (1967)].
[10] A. G. Litvak and V. I. Talanov, Izv. VUZov Radiofizika 10, 539 (1967).
[11] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Zh. Eksp. Teor. Fiz. 50, 1537 (1966) [Sov. Phys.-JETP 23, 1025 (1966)].
[12] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Paper at Session of Division of General and Applied Physics, USSR Academy of Sciences, May 1967.

*Formula (3) does not take into account the "fine structure" of the line in intervals on the order of T^{-1}.

LARGE-ANGLE ELASTIC SCATTERING

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Experimental data on the elastic scattering of protons by protons [1,2] and pions by protons [2] at high energy offer evidence that the decrease of the differential cross sections with increasing angle becomes weaker outside the region of the diffraction cone. This fact