the form of envelope shock waves [4,5]. We note that in the range $\lambda \sim 0.5 - 1\ \mu$ we have for the customarily employed liquids $\kappa < 0$; thus $\kappa \approx -0.07$ and $\kappa = -0.14$ for benzene and carbon disulfide, respectively, at $\lambda \sim 0.7\ \mu$ and a temperature 18 - 20°C. For the pulse parameters indicated above we have $z_2 \gtrsim 100$ cm, and (1) is applicable practically at all times, but if some fast processes are present in the initial pulse (including interaction with Raman-scattering components) it is necessary to take into account the variation of the amplitude envelope. We note that the change in the wave spectrum can be strongly asymmetrical here.

The foregoing has pertained to a plane wave, but for a channel with fixed (quasistationary) transverse field distribution we can draw similar conclusions. Thus, using the results of [9, 11] for a symmetrical three-dimensional channel and neglecting the relaxation time of the medium, we can easily obtain formula (1), where $n'(\xi)$ is equal to half the value of n' on the channel axis.

It is possible that the mechanism considered here explains to some degree the strong broadening of the spectrum of optical radiation in self-focusing, which was observed in some experiments [12]. We note also that the presence of a reactive nonlinearity in the resonator [8] (for example, due to the presence of nonlinear filters), can lead to a noticeable (albeit weaker than in self-focusing) broadening of the laser emission spectrum, especially for single-mode lasers.

In conclusion, I am grateful to A. V. Gaponov for discussion of the problems considered here.

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*Formula (3) does not take into account the "fine structure" of the line in intervals on the order of T^{-1}.

LARGE-ANGLE ELASTIC SCATTERING

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Experimental data on the elastic scattering of protons by protons [1,2] and pions by protons [2] at high energy offer evidence that the decrease of the differential cross sections with increasing angle becomes weaker outside the region of the diffraction cone. This fact was explained to be a consequence of the unitarity condition in [3,4], where it was found that outside the diffraction cone the behavior of the amplitude is given by the formula $A \sim e^{-\beta\sqrt{t}/2}$ (t is the square of the 4-momentum transfer). The coefficient β can be expressed (after correcting the misprints) in the form $\beta = \sqrt{8\alpha \ln(2\pi\alpha/\sigma_{in})}$, where σ_{in} is the cross section for the inelastic processes. Under less stringent assumptions than made in [3,4] we obtain from the unitarity condition an expression for the imaginary part of the elastic amplitude, in the form $\mathrm{Im}\, A \sim \exp(-bp\theta/2)$, where p and θ are the momentum and the scattering angle in the c.m.s., and the value of b differs greatly from β and agrees better with experiment.

We write the unitarity condition in the form:

$$\mathrm{Im}\, A(p,\theta) = \frac{1}{32\pi^2} \iint d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 A(p,\theta_1) A^*(p,\theta_2)}{\sqrt{(\cos\theta - \cos(\theta_1+\theta_2))(\cos(\theta_1-\theta_2) - \cos\theta)}} + F(p,\theta). \quad (1a)$$

Here F(p, θ) is the contribution of the inelastic processes to the elastic-scattering amplitude; the integration region is given by the conditions

$$|\theta_1 - \theta_2| \le \theta, \quad \theta \le \theta_1 + \theta_2 < 2\pi - \theta. \quad (1b)$$

If we assume that the elastic-scattering amplitude is pure imaginary in the entire region under consideration and that the contribution of the inelastic processes is in the form $F \sim \exp(-\alpha t/2)$, and we seek the solution of (1) by iterating this contribution, with a large number of iterations, $n_{eff} \gg 1$, then we arrive at the expression $A \sim \exp(-\beta\sqrt{t}/2)$ obtained in [3,4]. This solution, however, is subject to doubt, since it has turned out that the condition $n_{eff}^2 \ll \alpha p^2\theta^2$ is violated during its derivation; this condition is necessary in order for the iteration series to satisfy the unitarity condition.

We shall consider the unitarity condition directly for angles θ outside the diffraction cone: $\theta \gg \theta_d$. Using a reasonable approximation for the amplitudes $A(p, \theta_1)$ and $A^*(p, \theta_2$, based on experimental data on the differential cross sections (see below), we shall verify that the main contribution to the integral (1a) is made by two small symmetrical regions of angles from (1b): (i) $\theta_1 \le \theta_d \ll \theta$, $\theta_2 \sim \theta$ and (ii) $\theta_2 \le \theta_d \ll \theta$, $\theta_1 \sim \theta$. Namely, assuming that in the region of the diffraction cone the amplitude is pure imaginary, we can write it in the form $A(p, \theta) = 4ip^2\sigma_t \exp(-ap^2\theta^2)$, $\theta \le \theta_d$. Substituting this expression in (1a) for small θ_1 and θ_2, we get:

$$\mathrm{Im}\, A(p,\theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi a}} \int_{-\infty}^{\infty} d\nu \exp(-ap^2(\theta-\nu)^2/2)\, \mathrm{Im}\, A(p,\nu) + F(p,\theta). \quad (2)$$

We note that in order for our statements concerning the significant region of angles in the integral (1a) to be valid, the following relation must be satisfied in the transition from (1) to (2): $ap^2\theta^2 \gg 1$ and $ap^2\theta^2 \gg bp\theta$ (for the value of b see formula (3)).

The general solution of (2) is

$$\operatorname{Im} A(p,\theta) = F(p,\theta) + \frac{\sigma_t}{8\pi^2 a} \int_{-\infty}^{\infty} d\nu\, F(p,\nu) \int_{-\infty}^{\infty} du \frac{\exp[-(u^2/2ap^2) - iu(\theta-\nu)]}{1-(\sigma_t/4\pi a)\exp(-u^2/2ap^2)} + \tag{3}$$

$$+ C_1 \exp(-bp\theta/2) + C_2 \exp(bp\theta/2)$$

or approximately

$$\operatorname{Im} A(p,\theta) \cong F(p,\theta) + (2ap/b) \int_0^{\infty} d\nu [F(p,\theta+\nu) + F(p,\theta-\nu)] \exp(-bp\nu/2) + \tag{4}$$

$$+ C_1 \exp(-bp\theta/2) + C_2 \exp(bp\theta/2),$$

where

$$b = \sqrt{8a \ln(4\pi a/\sigma_t)}, \tag{5}$$

and C_1 and C_2 are undetermined constants. The experimental data on the differential elastic-scattering cross sections indicate that $C_2 = 0$.

The weak dependence of the elastic cross sections on the angle in the region of large angles (near $\pi/2$) gives grounds for assuming that Im A(p, θ) also depends weakly on the angle in this region. Then, according to (2), we have here:

$$\operatorname{Im} A(p,\theta) \cong F(p,\theta)/(1 - \frac{\sigma_t}{4\pi a}) . \tag{6}$$

The solution of the homogeneous equation $C_1\exp(-bp\theta/2)$ may fall in the region of intermediate angles. In this case the parameter for scattering through such angles (the quantity b) is expressed, as we see, in terms of the parameter of the diffraction cone (the quantity a) and the total cross section σ_t.

We have calculated the value of the constant b by means of formula (5) at those energies for which data are available concerning the quantity a [5-7]. The results are listed in Table 1. We see that with increasing pθ, in this region of angles, the slowest to decrease should be the differential cross sections of pp and $\bar{p}p$ scattering, followed by a faster rate (and approximately in the same manner within the limits of errors) by the cross sections for $\pi^{\pm}p$ and $K^{+}p$ scattering, and still faster the cross section for $K^{-}p$ scattering.

Knowing the value of b, we can find the contribution of the imaginary part to the differential cross section for scattering in the region of intermediate angles and to compare it with the available data [2] (see Table 2). The experiemntal errors, both in the determination of a and in the measurement of the cross sections for the scattering through such angles are still very large. It is therefore difficult to speak of a detailed agreement or disagreement between the results and experiment. It is seen, however, that whereas pp scattering can be satisfactorily described by the formula $C_1\exp[(-b/2)p\theta]$ (at angles not too close to $\pi/2$), for πp scattering the theoretical values differ significantly from the experimental ones. We can point to two possible causes of this discrepancy: either the value of pθ is not large enough for the angles and energies listed in the table, or else it is necessary to take into account

Table 1

Process	p_0, GeV/c	6.8	8,8	10,8	12,8	14.8	16,7	
π^+p	$b(\mathrm{GeV/c})^{-1}$	6.06 ± 0,23	6,29 ± 0,29	6,05 ± 0,30	6,61 ± 0,32	6,75 ± 0.36	6.78 ± 0,40	
π^-p	p_0	7,0	8.9	10.8	13,0	15,0	17,0	18,9
	b	6,18 ± 0,31	6,26 ± 0,28	6,58 ± 0,30	7,01 ± 0,32	7.32 ± 0,35	6,55 ± 0,44	7,76 ± 0,90
pp	p_0	6,8	8,8	10,8	12,8	14,8	16,7	19,6
	b	3,83 ± 0,21	3,72 ± 0,42	3,99 ± 0,43	4.55 ± 0.48	4.83 ± 0,54	4,25 ± 0,56	5.01 ± 0,60
$p\bar{p}$	p_0	7,2	8,9	10,0	12,0			
	b	3,64 ± 0,79	3.88 ± 0.63	2,88 ± 2,88	4,49 ± 0,63			
K^+p	p_0	6,8	9,8	12,8	14,8			
	b	5,34 ± 1,0	5,38 ± 0,60	6,18 ± 0,50	6,35 ± 0,50			
K^-p	p_0	7,2	9,0					
	b	7,77 ± 1,19	8,31 ± 1,0					

Table 2*

Process	p_0, Gev/c	$\cos\theta$	0,6630	0,6256	0,5926	0,5507	0,4758
pp	8	$(d\sigma/d\Omega)_{exp}$, mb/sr	5.34 ± 0,85	5,30 ± 0,63	3,69 ± 0,62	2.41 ± 0.33	1,64 ± 0,27
		$(d\sigma/d\Omega)_{theor}$, mb/sr	7,13 ± 0,43	5,09 ± 0,45	3,81 ± 0,43	2.68 ± 0,43	1.47 ± 0.30
pp	12	$\cos\theta$	0,8559	0,8079	0,7695	0,7599	
		$(d\sigma/d\Omega)_{exp}$	14,3 ± 1,4	8.81 ± 1.2	3,64 ± 0.91	3,30 ± 1,10	
		$(d\sigma/d\Omega)_{theor}$	16,0 ± 0,9	6,9 ± 1,0	3.70 ± 0,90	3,30 ± 0,80	
$\pi^{\pm}p$	8	$\cos\theta$	0,7881	0,7175	0,6469	0,5763	0,4350
		$(d\sigma/d\Omega)_{exp}$	17.0 ± 2,0	3,05 ± 0,73	0,23 ± 0,07	0,073 ± 0,022	0,063 ± 0,042
		$(d\sigma/d\Omega)_{theor}$	14,78 ± 0,64	4,23 ± 0,45	1,36 ± 0,22	0,47 ± 0.11	0,07 ± 0,03
π^-p	12	$\cos\theta$	0,8614	0,8152	0,7783	0,6767	
		$(d\sigma/d\Omega)_{exp}$	12,1 ± 1,0	1,35 ± 0,26	0,21 ± 0,07	< 0,06	
		$(d\sigma/d\Omega)_{theor}$	8,55 ± 0,34	2,27 ± 0,23	0,88 ± 0,13	0,09 ± 0,02	
π^+p	12	$\cos\theta$	0,8614	0,8152	0,7783	0,6767	
		$(d\sigma/d\Omega)_{exp}$	5,95 ± 1,1	0,88 ± 0,41	0,52 ± 0,21	–	
		$(d\sigma/d\Omega)_{theor}$	5,43 ± 0,23	1,55 ± 0,16	0,63 ± 0,10	0,07 ± 0,02	

the real part of the amplitude in the region of the diffraction cone.

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*The point with the largest cosθ in [2] was used for the normalization.

DOMINANCE OF THE ρ AND A_1 MESONS IN THE $\pi \to e\nu\gamma$ DECAY

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In an interesting paper [1], Weinberg found, assuming that the main contribution to the spectral function of the vector current is made by the ρ meson, and that of the axial current by the A_1 meson and the pion, that the ratio of the masses of the A_1 and ρ mesons is equal to $\sqrt{2}$, in splendid agreement with experiment. He used also the commutation relations of the current algebra and the conservation of the axial current (neglecting the pion mass).

We shall show that the same assumptions make it possible to find the axial part of the amplitude of the $\pi \to e\nu\gamma$ decay. The result apparently agrees with the experimental data [2]. Since all the momenta in the decay are of the order of the pion mass, the latter cannot be neglected. We shall therefore use the hypothesis of the partial conservation of the axial current.

It is convenient to separate from the axial part of the amplitude the term M^{ℓ} corresponding to the emission of a γ quantum by an electron, and also the term M^{π} containing the emission of a γ quantum by a pion as well as terms of zero power in the γ-quantum momentum (contact diagram). Then the matrix element of the decay $\pi^+ \to e\nu\gamma$ is represented by a sum of four parts

$$M = i\frac{G}{\sqrt{2}}\sqrt{4\pi\alpha}\,\phi\,\epsilon_\mu [\, M^{\ell}_{\mu} + i_\nu (M^{\pi}_{\mu\nu} + M^{A}_{\mu\nu} + M^{V}_{\mu\nu})], \tag{1}$$

$$M^{\ell}_{\mu} = f\,\bar{u}_{\ell}\Big[\frac{(2\ell - k)_\mu}{2\ell k - k^2} - \frac{\sigma_{\mu\nu}k_\nu}{2\ell k - k^2}\Big]\hat{q}(1+\gamma_5)\,u_\nu, \tag{2}$$

$$M^{\pi}_{\mu\nu} = f\Big[-g_{\mu\nu} + \frac{(2p+k)_\mu p_\nu}{p^2 - \mu^2}\Big], \tag{3}$$

$$M^{V}_{\mu\nu} = -i\,a\,\epsilon_{\mu\nu\lambda\sigma}k_\lambda p_\sigma, \tag{4}$$

$$M^{A}_{\mu\nu} = b(g_{\mu\nu}(kp) - p_\mu k_\nu) + c(g_{\mu\nu}k^2 - k_\mu k_\nu). \tag{5}$$