the real part of the amplitude in the region of the diffraction cone.

The authors are grateful to I. I. Roizen, E. L. Feinberg, and D. S. Chernavskii for a discussion of the work.

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*The point with the largest cosθ in [2] was used for the normalization.

DOMINANCE OF THE ρ AND A_1 MESONS IN THE $\pi \to e\nu\gamma$ DECAY

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Submitted 1 August 1967
ZhETF Pis'ma 6, No. 8, 815-817 (15 October 1967)

In an interesting paper [1], Weinberg found, assuming that the main contribution to the spectral function of the vector current is made by the ρ meson, and that of the axial current by the A_1 meson and the pion, that the ratio of the masses of the A_1 and ρ mesons is equal to $\sqrt{2}$, in splendid agreement with experiment. He used also the commutation relations of the current algebra and the conservation of the axial current (neglecting the pion mass).

We shall show that the same assumptions make it possible to find the axial part of the amplitude of the $\pi \to e\nu\gamma$ decay. The result apparently agrees with the experimental data [2]. Since all the momenta in the decay are of the order of the pion mass, the latter cannot be neglected. We shall therefore use the hypothesis of the partial conservation of the axial current.

It is convenient to separate from the axial part of the amplitude the term M^{ℓ} corresponding to the emission of a γ quantum by an electron, and also the term M^{π} containing the emission of a γ quantum by a pion as well as terms of zero power in the γ-quantum momentum (contact diagram). Then the matrix element of the decay $\pi^+ \to e\nu\gamma$ is represented by a sum of four parts

$$M = i\frac{G}{\sqrt{2}}\sqrt{4\pi\alpha}\,\phi\,\epsilon_\mu[\,M^{\ell}_{\mu} + i_\nu(M^{\pi}_{\mu\nu} + M^{A}_{\mu\nu} + M^{V}_{\mu\nu})], \qquad (1)$$

$$M^{\ell}_{\mu} = f\,\bar{u}_{\ell}\left[\frac{(2\ell - k)_\mu}{2\ell k - k^2} - \frac{6_{\mu\nu}k_\nu}{2\ell k - k^2}\right]\hat{q}(1+\gamma_5)\,u_\nu, \qquad (2)$$

$$M^{\pi}_{\mu\nu} = f\left[-g_{\mu\nu} + \frac{(2p+k)_\mu p_\nu}{p^2 - \mu^2}\right], \qquad (3)$$

$$M^{V}_{\mu\nu} = -i\,a\,\epsilon_{\mu\nu\lambda\sigma}k_\lambda p_\sigma, \qquad (4)$$

$$M^{A}_{\mu\nu} = b(g_{\mu\nu}(kp) - p_\mu k_\nu) + c(g_{\mu\nu}k^2 - k_\mu k_\nu). \qquad (5)$$

Here ε_μ is the γ-quantum polarization vector, $j_\mu = \bar{u}_\ell \gamma_\mu (1 + \gamma_5) u_\nu$, q is the pion momentum, k that of the photon, ℓ that of the electron, p the total momentum of the lepton pair, and μ the pion mass. The constant f is connected with the lifetime of the π^+ meson, and the constant a is expressed [3] in terms of the lifetime of the π^0 meson. (The assumed T-invariance makes it possible to regard f, a, b, and c as real quantities [3].)

$$\frac{1}{\tau_{\pi+}} = \frac{G^2 f^2 m_\mu^2 \mu}{8\pi} \left(1 - \frac{m_\mu^2}{\mu^2}\right)^2, \quad \frac{1}{\tau_{\pi^0}} = \frac{\pi}{2} a^2 \alpha^2 \mu^3. \tag{6}$$

In a real decay we have $k^2 = 0$, and the second term in (5) does not make any contribution. We shall need $k^2 \neq 0$ to determine b.

Let us consider the matrix element of the axial current, which is equal to the sum $M^\pi_{\mu\nu} + M^A_{\mu\nu}$

$$M^\pi_{\mu\nu} + M^A_{\mu\nu} = \int dx dy \exp(ipx - iqy)(\Box_y - \mu^2) \cdot \langle 0 | T\{ v^0_\mu(o) \cdot a^-_\nu(x) \phi^+(y)\} | 0 \rangle. \tag{7}$$

v^0_μ and a^-_ν are the operators of the vector and axial currents (only the isovector part of the electromagnetic current makes a contribution), and we have used a reduction formula with respect to the pion. With the aid of the usual commutation relations and the hypothesis of partial conservation of the axial current ($\partial_\mu a^\pm_\mu = \mu^2 f \phi^\pm$), it can be shown that $p_\nu M^A_{\mu\nu} = 0$ for arbitrary k^2, p^2, and q^2, and consequently $c = 0$.

We replace in (7) the pion field by the divergence of the axial current and put $q = 0$. Then

$$M^\pi_{\mu\nu} + M^A_{\mu\nu} \xrightarrow[q \to 0]{} -\frac{2i}{f} \int dx e^{ikx} \langle o | T\{ a^0_\mu(x) a^0_\nu(o) - v^0_\mu(x) v^0_\nu(o)\} | o \rangle. \tag{8}$$

Following Weinberg [1], we assume that the ρ meson makes the main contribution in the vector current, and the A_1 meson and pion in the axial current. This yields

$$M^\pi_{\mu\nu} + M^A_{\mu\nu} \xrightarrow[q \to 0]{} f\left\{ \frac{k_\mu k_\nu}{k^2 - \mu^2} - \frac{2m_\rho^2}{k^2 - m_A^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right) + \frac{2m_\rho^2}{k^2 - m_\rho^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_\rho^2}\right)\right\}. \tag{9}$$

Since all the momenta are much smaller than the ρ-meson mass, we can, confining ourselves to a quadratic expansion, determine b:

$$b = -\frac{3}{2} \frac{f}{m_\rho^2}. \tag{10}$$

Using $\tau_{\pi^0} = 0.89 \times 10^{-16}$ sec [4], we get for the modulus of the ratio b/a ($f = 0.95\ \mu$)

$$|b/a| = 1.8. \tag{11}$$

From the experimental data [2] we can obtain two solutions for b/a (the accuracy with which the spectrum is measured is insufficient for a unique determination of b/a)

$$b/a = -2 \pm 0.1, \; b/a = 0.3 \pm 0.1$$

The number obtained by us agrees with the first solution.

In conclusion, I am grateful to V. V. Sokolov for a discussion of the work.

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Article by A. I. Vainshtein, Vol. 6, No. 8 (p. 266)

In the term $M^{\pi}_{\mu\nu}$, corresponding to emission of a γ quantum by a pion at $k^2 \neq 0$, no account was taken of the pion electromagnetic form factor $F_{\pi}(k^2)$. Allowance for this factor changes the coefficient b, which becomes equal to

$$b = -\frac{3}{2}\frac{f}{m_{\rho}^{2}} + 2f\,\frac{dF_{\pi}(k^2)}{dk^2}\Big|_{k^2=0}$$

which agrees with the result obtained in the simultaneously performed investigation of T. Das, V. S. Mathur, and S. Okubo (Phys. Rev. Lett. 19, 859 (1967). In determining $dF_{\pi}(k^2)/dk^2$, these authors neglected the form factor of the $\rho\pi\pi$ vertex, which apparently contradicts the experimental width ratio of the A_1 and ρ mesons. If this ratio is assumed equal to unity, then we get $|\gamma| \approx 0.3$. The value given in our paper is incorrect.