which the spectrum is measured is insufficient for a unique determination of b/a)

$$b/a = -2 \pm 0.1, \quad b/a = 0.3 \pm 0.1$$

The number obtained by us agrees with the first solution.

In conclusion, I am grateful to V. V. Sokolov for a discussion of the work.

[1] S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
[2] P. Depommier et al., Phys. Lett. 7, 285 (1963).
[3] V. G. Vaks and B. L. Ioffe, Zh. Eksp. Teor. Fiz. 35, 221 (1958) [Sov. Phys.-JETP 8, 151 (1959)].
[4] A. H. Rosenfeld et al. Revs. Modern Phys. 39, 1 (1967).

CIRCULAR POLARIZATION OF PHOTONS IN THE PROCESS $e^+e^- \to 2\gamma$ AT HIGH ENERGIES

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We present in this paper the results of a calculation of the spin correlation in the process $e^+e^- \to 2\gamma$ (A); this correlation arises in transverse polarization of the electron and the positron as a result of the imaginary part of the corresponding amplitude. The imaginary part of the amplitude of the process (A) is connected with radiative corrections determined by the diagrams a and b of Fig. 1.

The correlation in question is of the form $\alpha^3\zeta_+\zeta_-\lambda(\vec{n}_\gamma\cdot\vec{\xi})\vec{n}_\gamma[\vec{n}_e \times \vec{\xi}]$ (B), where $\alpha = 1/137$, and $\vec{n}_e$ and $\vec{n}_\gamma$ are unit vectors along the momenta of the electron and of one of the gamma quanta; $\lambda = \pm 1$ is the helicity of this quantum ($\lambda\vec{n}_\gamma$ - circular polarization axial vector); $\zeta_+\vec{\xi} = \vec{\zeta}_+$ and $\zeta_-\vec{\xi} = \vec{\zeta}_-$ are the mean values of the spins of e^+ and e^- ($\vec{\xi}$ is a unit vector, $\vec{\xi} \perp \vec{n}_e$).

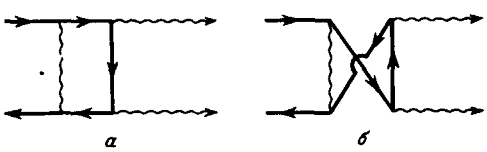

Fig. 1

As is well known [1] (see also [2]), the electrons and positrons in colliding-beam experiments should be polarized perpendicular to the orbit plane ($\vec{\xi} \perp \vec{n}_e$), and the experimental situation apparently corresponds to $\zeta_+ = -\zeta_-$ and $|\zeta_-| \approx 0.9$.

The correlation (B) is T-odd and is connected with the temporal irreversibility of the process (A), owing to the imaginary part in the amplitude. The correlation (B) is the only possible T-odd contribution to the cross section in the case of transverse and mutually parallel (or antiparallel polarization of the initial particles.

If only the polarization of one γ quantum is measured in the experiment, the circular polarization of this quantum also enters in an ordinary T-odd correlation $\sim\alpha^2\zeta\lambda(\vec{\xi}\cdot\vec{n}_e)$ (C). At high energies, however, this term of the differential cross section (in the case when e^+ and e^- are transversely polarized) should decrease like $1/\gamma$ compared with the correlation (B) ($\gamma = \varepsilon/m$, ε is the c.m.s. energy). This is connected with the conservation of the helicity of the electron in the electromagnetic vertices. Thus, at very high energies ($\gamma >> 1/\alpha$), the γ quantum in process (A) is circularly polarized only as a result of T-odd effects.

With allowance for the circular polarization of the quantum, the differential cross section of the process (A) (in the c.m.s.) is

$$\frac{d\sigma_\lambda}{d\Omega} = \frac{\alpha^2}{8m^2\gamma^2}\Big[\frac{2}{\sin^2\theta} - 1 - \zeta_+\zeta_-\Big(1 - \frac{2}{\sin^2\theta}(\vec n_\gamma\vec\xi\,)^2\Big) - \tag{1}$$

$$- \frac{2}{\gamma}\lambda(\zeta_+ + \zeta_-)\frac{\vec\xi\,\vec n_\gamma}{\sin^2\theta} - 4\alpha\zeta_+\zeta_-\lambda(\vec n_\gamma\vec\xi\,)\,\vec n_\gamma[\vec n_e\vec\xi\,]\frac{f(\theta)}{\sin^4\theta}\Big],$$

$$f(\theta) = \cos^2\frac{\theta}{2}\Big(1 + \frac{\ln\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\Big) - \sin^2\frac{\theta}{2}\Big(1 + \frac{\ln\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\Big) +$$

$$+ \frac{1}{4}\sin^2\theta\ln\mathrm{tg}^2\frac{\theta}{2}\,. \tag{2}$$

Formula (1) holds for $\gamma \gg 1$ and $\theta\gamma \gg 1$. The degree of circular polarization P is defined as

$$P = \frac{(d\sigma_+ - d\sigma_-)}{(d\sigma_+ + d\sigma_-)}\,.$$

With $\vec n_e\cdot\vec n_\gamma = \cos\theta$ and $\vec n_\gamma\cdot\vec\xi = \cos\phi\sin\theta$, we get for the degree of circular polarization (as $\gamma\to\infty$)

$$P(\theta,\phi) = -\alpha\zeta_+\zeta_-\frac{\sin 2\phi\, f(\theta)}{1 - \frac{1}{2}\sin^2\theta(1 - \zeta_+\zeta_-\cos 2\phi)}\,. \tag{3}$$

It is important that $P(\theta,\phi)$ does not depend on the energy. At large antiparallel polarizations of the electron and positron, the differential cross section has a minimum in the electron-polarization direction ($\theta = \pi/2$, $\phi = 0$ and π), leading to a small denominator in (3). For a specified θ the polarization P has a maximum in the direction given by

$$\mathrm{tg}^2\phi = \Big[1 - \frac{\sin^2\theta}{2}(1 - \zeta_+\zeta_-)\Big]\Big[1 - \frac{\sin^2\theta}{2}(1 + \zeta_+\zeta_-)\Big]^{-1}$$

At the maximum polarization ($|\zeta| \approx 92\%$) of the electrons and positrons (see [1]), the absolute maximum of P is reached at $\theta \approx 60^\circ$ and $\phi \approx 30^\circ$, and is equal to $P \approx 0.16\alpha$. The plot of $P(\theta)$ at $\phi = 30^\circ$ is shown in Fig. 2, which illustrates also the dependence of the differential cross section on θ. The low value of the polarization ($P \approx 0.1\alpha$) is somewhat unexpected.

It follows from (3) that there is no polarization when the γ quantum is produced in any of the three planes defined by the three mutually perpendicular vectors $\vec n_e$, $\vec\xi$, and $\vec n_e\times\vec\xi$.

The degree of circular polarization $P(\theta,\phi)$ is antisymmetrical with respect to reflections in the plane perpendicular to the electron momentum $\vec n_e$ (the function $f(\theta)$ reverses sign when

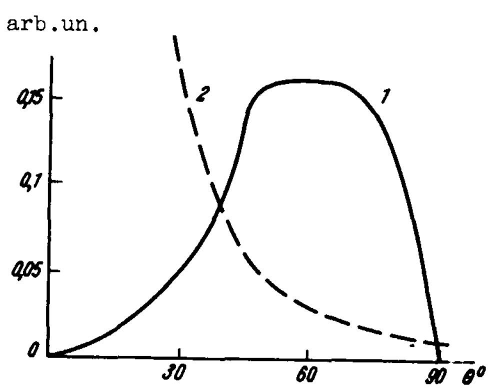

Fig. 2. Curve 1 - the function $-(1/\alpha)P(\theta,\phi)$ ($P(\theta,\phi)$ is the degree of circular polarization of the γ quantum, see formula (3) for $\phi = 30^\circ$, $\zeta_+ = -\zeta_-$, $|\zeta_-| = 0.92$; curve 2 - the function $\sum_{\lambda=\pm1} d\sigma_\lambda/d\Omega$ ($d\sigma_\omega/d\Omega$) is the differential cross section, see (1)).

$\theta \to \pi - \theta$). This is the consequence of the C-invariance of electrodynamics. If the correlation (B) were the result of a true violation of T- (and CP-) invariance in some bare interactions, then the polarization $P(\theta,\phi)$ would be, to the contrary, a symmetrical function relative to reflections in this plane.

In our opinion, an experimental investigation of the γ-quantum circular polarization in process (A) at high energies would be of interest from the point of view of checking on quantum electrodynamics, since this polarization is determined by the contribution of the higher approximations in the electromagnetic interaction. The existence of anomalous interactions of the type $(\vec{E}^2 - \vec{H}^2)^2$ could become manifest at very high energies in an appreciable change of the function $P(\theta,\phi)$ and in the appearance of an energy dependence of this function.

It is also of interest to check on the consequences of CP-invariance, although, of course, it is difficult to expect a noticeable manifestation of CP-odd interactions in this process. The organization of appropriate experiments is apparently fraught with great difficulties.

[1] I. Ternov, Yu. Loskutov, and L. Korovina, Zh. Eksp. Teor. Fiz. 41, 1294 (1961) [Sov. Phys.-JETP 14, 921 (1962). A. Sokolov and I. Ternov, Dokl. Akad. Nauk SSSR 153, 1052 (1963) [Sov. Phys.-Dokl. 8, 1203 (1964)].
[2] V. Baier and V. Fadin, Dokl. Akad. Nauk SSSR 161, 74 (1965) [Sov. Phys.-Dokl. 10,204(1965)].

GENERALIZATION OF SNYDER'S QUANTIZED SPACE-TIME THEORY

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We formulate a theory of quantized space-time, symmetrical with respect to the coordinate and momentum variables and admitting of a group-theoretical interpretation.

In Snyder's quantized space theory [1] and in theories of curved momentum space [2,3] the symmetry between the coordinate and momentum variables is strongly violated: the coordinates x_i and the operators of Lorentz rotations F_{ij} are interpreted as infinitesimal operators of the group of motions of four-dimensional momentum space of constant curvature. We shall show in this note how to avoid this asymmetry, by considering the coordinates x_i, the momenta p_i, the operators of Lorentz rotations F_{ij}, and the action I as infinitesimal operators of the group of motions of an unobservable six-dimensional space of definite signature. We postulate the following commutation relations between the 15 introduced operators (x_i, p_i, F_{ij}, and I):

$$[F_{lj}, F_{sk}] = i(g_{ls}F_{jk} - g_{js}F_{lk} - g_{lk}F_{js} + g_{jk}F_{ls}),$$

$$[F_{lj}, I] = 0,$$

$$[F_{lj}, x_s] = i(g_{ls}x_j - g_{js}x_l), \quad [F_{lj}, p_s] = i(g_{ls}p_j - g_{js}p_l), \qquad (1)$$

$$[I, x_s] = i\ell_0^2 g_{sk}x_k, \qquad [I, p_s] = -im_0^2 g_{sk}x_k,$$

$$[x_s, x_j] = i\ell_0^2 F_{sj}, \qquad [p_s, p_j] = im_0^2 F_{sj},$$

$$[p_s, x_j] = iIg_{sj}$$