$\theta \to \pi - \theta$). This is the consequence of the C-invariance of electrodynamics. If the correlation (B) were the result of a true violation of T- (and CP-) invariance in some bare interactions, then the polarization $P(\theta,\phi)$ would be, to the contrary, a symmetrical function relative to reflections in this plane.

In our opinion, an experimental investigation of the γ-quantum circular polarization in process (A) at high energies would be of interest from the point of view of checking on quantum electrodynamics, since this polarization is determined by the contribution of the higher approximations in the electromagnetic interaction. The existence of anomalous interactions of the type $(\vec{E}^2 - \vec{H}^2)^2$ could become manifest at very high energies in an appreciable change of the function $P(\theta,\phi)$ and in the appearance of an energy dependence of this function.

It is also of interest to check on the consequences of CP-invariance, although, of course, it is difficult to expect a noticeable manifestation of CP-odd interactions in this process. The organization of appropriate experiments is apparently fraught with great difficulties.

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GENERALIZATION OF SNYDER'S QUANTIZED SPACE-TIME THEORY

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We formulate a theory of quantized space-time, symmetrical with respect to the coordinate and momentum variables and admitting of a group-theoretical interpretation.

In Snyder's quantized space theory [1] and in theories of curved momentum space [2,3] the symmetry between the coordinate and momentum variables is strongly violated: the coordinates x_i and the operators of Lorentz rotations F_{ij} are interpreted as infinitesimal operators of the group of motions of four-dimensional momentum space of constant curvature. We shall show in this note how to avoid this asymmetry, by considering the coordinates x_i, the momenta p_i, the operators of Lorentz rotations F_{ij}, and the action I as infinitesimal operators of the group of motions of an unobservable six-dimensional space of definite signature. We postulate the following commutation relations between the 15 introduced operators (x_i, p_i, F_{ij}, and I):

$$[F_{lj}, F_{sk}] = i(g_{ls}F_{jk} - g_{js}F_{lk} - g_{lk}F_{js} + g_{jk}F_{ls}),$$

$$[F_{lj}, I] = 0,$$

$$[F_{lj}, x_s] = i(g_{ls}x_j - g_{js}x_l), \quad [F_{lj}, p_s] = i(g_{ls}p_j - g_{js}p_l), \tag{1}$$

$$[I, x_s] = i\ell_0^2 g_{sk}x_k, \qquad [I, p_s] = -im_0^2 g_{sk}x_k,$$

$$[x_s, x_j] = i\ell_0^2 F_{sj}, \qquad [p_s, p_j] = im_0^2 F_{sj},$$

$$[p_s, x_j] = iIg_{sj}$$

ℓ_0 is the elementary length, m_0 the elementary momentum ($\hbar = c = 1$); in the classical limiting case ($\ell_0 \to 0$, $m_0 \to 0$) it is necessary to stipulate $I \to 1$ ($g_{11} = g_{22} = g_{33} = -g_{44} = 1$).

The commutation relations (1) are isomorphic to the commutation relations for the infinitesimal operators of the rotation group O(p, q) of six-dimensional space, namely O(1,5) for $\ell_0^2 > 0$, $m_0^2 > 0$; O(2,4) for $\ell_0^2 > 0$, $m_0^2 < 0$ or $m_0^2 > 0$, $\ell_0^2 < 0$; and O(3,3) for $\ell_0^2 < 0$, $m_0^2 < 0$.

Each irreducible representation of the group of six-dimensional rotations is determined uniquely by specifying the eigenvalues of three Casimir operators K_α^6, whose form in terms of the dimensionless operators T_{ij} ($1 \leq i, j \leq 6$) is

$$K_1^6 = \epsilon_{ijs\ell mn} T_{ij} T_{s\ell} T_{mn}, \; K_2^6 = (T_{ij})^2, \; K_3^6 = (\epsilon_{ijslmn} T_{se} T_{mn})^2,$$

where $\varepsilon_{ijs\ell mn}$ is a completely antisymmetrical tensor of rank six, and

$$T_{56} = 1/m_0\ell_0, \; T_{6a} = x_a/\ell_0, \; T_{5a} = p_a/m_0, \; T_{a\beta} = F_{a\beta}$$
$$(1 \leqslant a, \beta \leqslant 4).$$

In the "classical" limiting case $(m_0\ell_0)^2 K_2^6 \to \pm I^2$ it is therefore sufficient to require $K_2^6 = \pm(1/m_0\ell_0)^2$ (depending on the sign with which $(1/m_0\ell_0)$ enters in K_2^6) in order to ensure a correct transition to the limit. For K_1^6 and K_3^6 we get the following limiting expressions:

$$K_1^6 (m_0\ell_0) \to \epsilon_{\alpha\beta\gamma\delta} S_{\alpha\beta} S_{\gamma\delta}, \; K_3^6 (m_0\ell_0)^2 \to (S_{\alpha\beta})^2, \tag{2}$$

where $S_{\alpha\beta} \equiv F_{\alpha\beta} - (x_\alpha p_\beta - p_\alpha x_\beta)$, and $\varepsilon_{\alpha\beta\gamma\delta}$ is a perfectly antisymmetrical tensor of fourth rank. It follows from (2) that, apart from coefficients, K_1^6 and K_3^6 go over into two Casimir operators of the proper Lorentz group. Thus, three Casimir operators specify the irreducible representation of the group of six-diemnsional rotations, describing in the "classical" limiting case a state that transforms in accordance with a definite irreducible representation of the proper Lorentz group.

The space-motion operators P_i and F_{ij} form a five-dimensional subgroup of the six-dimensional group of rotations, with Casimir operators

$$K_1^5 = (\epsilon_{ij\ell ms} T_{j\ell} T_{ms})^2, \; K_2^5 = (T_{ij})^2 \quad (1 \leqslant i, j \leqslant 5), \tag{3}$$

which go over in the "classical" limiting case into the square of the mass and the square of the invariant spin. Therefore the problem of determining the spectrum of the masses and of the spins reduces in this variant of the theory of quantized space to finding the eigenvalues of the Casimir operators of the subgroup of five-dimensional rotations for a specified representation of a group of six-dimensional rotations. This problem has not been solved yet.

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DIFFERENCE BETWEEN THE SPECTRA OF $K^0_{\mu 3}$ and $\bar{K}^0_{\mu 3}$ DECAYS

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We derive below an expression for the difference between the differential probabilities of $K^0_{\mu 3}$ and $\bar{K}^0_{\mu 3}$ decays; this difference should occur if CP-invariance is violated in these decays, and is of the order of $\alpha \mathrm{Im}\xi$; the total probabilities of the $K^0_{\mu 3}$ and $\bar{K}^0_{\mu 3}$ decays are the same in this approximation.

1.We consider first the question of the possible difference between the probabilities $d^2\Gamma$ and $d^2\bar{\Gamma}$ (spectra and angular distributions) of the decays*

$$K^0 \to \mu^+ \nu \pi^- \tag{1}$$

and

$$\bar{K}^0 \to \mu^- \bar{\nu} \pi^+. \tag{2}$$

Such a difference should arise if CP-invariance is violated in the decays (1) and (2) ($\mathrm{Im}\xi \neq 0$). In addition, in order for such a difference to arise it is necessary that an interaction take place in the final state, in our case electromagnetic interaction between the muon and the pion.

2. We write down the amplitude of the decay (1) without allowance for the electromagnetic interaction in the form

$$V = p_k + \chi p_\mu, \tag{3}$$

where p_k (p_μ) is the 4-momentum of the K-meson (muon), and $2\chi = \xi - 1$. When account is taken of the electromagnetic interaction between the pion and the muon, the amplitude (3) takes the form

$$V' = (1 + \phi_1 + \chi\phi_2) p_K + (\chi + \chi\psi_1 + \psi_2) p_\mu, \tag{4}$$

where ϕ_1, ϕ_2, ψ_1, and ψ_2 are radiative corrections on the order of $\alpha = 1/137$. From (4) we readily obtain

$$\frac{d^2\bar{\Gamma} - d^2\Gamma}{d^2\Gamma} = 4\,\mathrm{Im}\,\chi \frac{\mathrm{Im}\,\phi_2 F_k - \mathrm{Im}\,\psi_2 F_\mu + \mathrm{Im}(\psi_1 - \phi_1) F_{K\mu}}{F_K + \mathrm{Re}\,\chi F_{K\mu} + |\chi|^2 F_\mu}, \tag{5}$$

where

$$F_K = 2(p_K p_\mu)(p_K p_\nu) - p_K^2 (p_\mu p_\nu), \tag{6}$$

$$F_\mu = m_\mu^2 (p_\mu p_\nu), \tag{7}$$

$$F_{K\mu} = 2m_\mu^2 (p_K p_\nu). \tag{8}$$

The imaginary parts of the radiative corrections that enter in (5) were calculated in [1], where it was shown that if the form factors of the weak vertex are neglected, we get