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DIFFERENCE BETWEEN THE SPECTRA OF $K^0_{\mu3}$ and $\bar{K}^0_{\mu3}$ DECAYS

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We derive below an expression for the difference between the differential probabilities
of $K^0_{\mu3}$ and $\bar{K}^0_{\mu3}$ decays; this difference should occur if CP-invariance is violated in these de-
cays, and is of the order of $\alpha \mathrm{Im}\xi$; the total probabilities of the $K^0_{\mu3}$ and $\bar{K}^0_{\mu3}$ decays are the
same in this approximation.

1. We consider first the question of the possible difference between the probabilities
$d^2\Gamma$ and $d^2\bar{\Gamma}$ (spectra and angular distributions) of the decays*

$$K^0 \rightarrow \mu^+ \nu \pi^- \tag{1}$$

and

$$\bar{K}^0 \rightarrow \mu^- \bar{\nu} \pi^+. \tag{2}$$

Such a difference should arise if CP-invariance is violated in the decays (1) and (2) ($\mathrm{Im}\xi \neq 0$).
In addition, in order for such a difference to arise it is necessary that an interaction take
place in the final state, in our case electromagnetic interaction between the muon and the
pion.

2. We write down the amplitude of the decay (1) without allowance for the electromag-
netic interaction in the form

$$V = P_k + \chi p_\mu, \tag{3}$$

where p_k (p_μ) is the 4-momentum of the K-meson (muon), and $2\chi = \xi - 1$. When account is taken
of the electromagnetic interaction between the pion and the muon, the amplitude (3) takes the
form

$$V' = (1 + \phi_1 + \chi\phi_2)\, p_K + (\chi + \chi\psi_1 + \psi_2)\, p_\mu, \tag{4}$$

where ϕ_1, ϕ_2, ψ_1, and ψ_2 are radiative corrections on the order of $\alpha = 1/137$. From (4) we
readily obtain

$$\frac{d^2\bar{\Gamma} - d^2\Gamma}{d^2\Gamma} = 4\,\mathrm{Im}\,\chi \,\frac{\mathrm{Im}\,\phi_2 F_k - \mathrm{Im}\,\psi_2 F_\mu + \mathrm{Im}(\psi_1 - \phi_1) F_{K\mu}}{F_K + \mathrm{Re}\,\chi F_{K\mu} + |\chi|^2 F_\mu}, \tag{5}$$

where

$$F_K = 2(p_K p_\mu)(p_K p_\nu) - p_K^2 (p_\mu p_\nu), \tag{6}$$

$$F_\mu = m_\mu^2 (p_\mu p_\nu), \tag{7}$$

$$F_{K\mu} = 2m_\mu^2 (p_K p_\nu). \tag{8}$$

The imaginary parts of the radiative corrections that enter in (5) were calculated in [1],
where it was shown that if the form factors of the weak vertex are neglected, we get

$$\text{Im}\psi_1 = \text{Im}\phi_1 , \qquad (9)$$

$$\text{Im}\phi_2 = \frac{m_\mu^2}{p^2} \text{Im}\psi_2 , \qquad (10)$$

$$\text{Im}\psi_2 = \frac{\alpha}{2} \frac{p_\pi p_\mu + m_\pi^2}{\sqrt{q^2 p^2}} \left(1 - \frac{r^2 q^2}{3}\right) , \qquad (11)$$

where r is the electromagnetic radius of the pion, and

$$p^2 = (p_\pi + p_\mu)^2 , \quad q^2 p^2 = [(p_\mu p_\pi)^2 - m_\mu^2 m_\pi^2] .$$

It follows from (5) - (11) that

$$\frac{d^2\bar{\Gamma}}{d^2\Gamma} - 1 = 2\alpha \text{Im}\chi \frac{m_\mu^2}{p^2} \frac{p_\pi p_\mu + m_\pi^2}{\sqrt{q^2 p^2}} (1 - \frac{r^2 q^2}{3}) \frac{F_K - p^2(p_\mu p_\nu)}{F_K + \text{Re}\chi F_{K\mu} + |\chi|^2 F_\mu} . \qquad (12)$$

3. If we integrate the numerator of (5) with respect to the direction of the relative momentum of the pion and muon at a fixed mass of the pion-muon system, then it can be readily verified that the result is equal to zero.** Thus, the neutrino spectra in the decays (1) and (2) are the same. The reason is that the states with different I^P are not mixed in the pion-muon system by the interaction between the pion and the muon, but merely acquire different phases. Since the entire effect of the inequality of the differential probabilities of decays (1) and (2) is due to interference between states having different I, the orthogonality of these states should cause the effect to vanish upon integration with respect to the angle of muon emission in the pion-muon c.m.s. This reasoning holds also when account is taken of the form factors of the weak vertex. It is obvious that the total widths of decays (1) and (2) should also be the same in the first nonvanishing approximation in α. The latter conclusion can be arrived at directly by starting from the CPT theorem and recognizing that the "leakage" in the $K^0 \to \mu^+ \nu \pi^- \gamma$ and $\bar{K}^0 \to \mu^- \bar{\nu}\pi^+ \gamma$ channels is connected with $\mu\pi \to \mu\pi\gamma$ transitions on the mass shell and is therefore a quantity on the order of α^2 [2].

It follows from (12) that in that region of the Dalitz diagram where the differential decay probability is large (high-energy pions, $F_K \gg F_\mu$, $F_{K\mu}$) we have $d^2\bar{\Gamma}/d^2\Gamma - 1 \simeq 10^4$ (if Im$\xi \simeq 0.1$). (We recall that the most exact experiment [3] yields Im $= 0.014 \pm 0.066$.) In the region of relatively low differential decay probability (where $F_K \simeq F_\mu$) the value of $d^2\bar{\Gamma}/d^2\Gamma$ - 1 may be on the order of 10^{-3}.

The discussed difference in the spectra of the decays (1) and (2) could become manifest, in principle, in experimental measurements of the charge asymmetry $\Gamma(K_L^0 \to \mu^+ \nu \pi^-)/\Gamma(K_L^0 \to \mu^- \bar{\nu}\pi^+)$ if the particle registration efficiency is different in different sections of the spectrum. It follows from the foregoing that if Im$\xi \leq 0.1$, then at an experimental accuracy $\sim 10^{-3}$ the effect under discussion is insignificant, but might appear at an accuracy on the order of 10^{-4}.

4. We note that for the decays K_{e3}^0 and $\bar{K}_{e3}^0$ the discussed difference in the spectra should be negligibly small (see (12)).

Different spectra but equal total partial widths should be also obtained when CP-invari-

ance is violated in the decays $K^0 \rightarrow e^+ \nu \pi^- \pi^0$ and $\bar{K}^0 \rightarrow e^- \bar{\nu} \pi^+ \pi^0$. On the other hand, in the decays $K^+ \rightarrow e^+ \nu \pi^+ \pi^-$ and $K^- \rightarrow e^- \bar{\nu} \pi^- \pi^+$ CP violation should lead to differences not only in the differential probabilities, but also in the partial widths of the decays (owing to "leakage" in the $K^+ \rightarrow e^+ \nu 2\pi^0$ and $K^- \rightarrow e^- \bar{\nu} 2\pi^0$ channels). These effects will not be small if the $\Delta T = 1/2$ rule is violated in K_{e4} decay.

The author is grateful to I. Yu. Kobzarev for useful critical discussions and to V. I. Zakharov who called his attention to the role of the $\Delta T = 1/2$ rule in K_{e4} decays.

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*We assume that the rule $\Delta Q = \Delta S$ is valid.

**In this integration, expressions of the type $(p_\pi + p_\mu)^2$ are taken outside the integral sign, and the integration in the remaining expression reduces essentially to the substitution $p_\mu \rightarrow p_K - p_\nu$.

DISLOCATIONS IN AN ANISOTROPIC MEDIUM

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Calculations of the elastic fields produced by dislocations in crystals serve as the basis for many applications of dislocation theory. However, allowance for the elastic anisotropy of the crystal can be made only in the case of straight-line dislocations, when the problem reduces to a planar one and admits the use of complex-variable methods (see [1]). The field of curvilinear dislocations is usually estimated very roughly in the elastic-isotropic approximation.

For a dislocation loop Γ with a Burgers vector b in an unbounded anisotropic medium, the displacement $u_k(x)$ coincides, in accord with the reciprocity theorem, with the work done by the stresses σ_{ij}^k produced by a unit force applied at the point x in the direction k on the discontinuities of the displacements b_j, needed to form the dislocation [2,3]

$$u_k(x) = \int b_i \sigma_{ij}^k (x^1 - x) dS_j. \tag{1}$$

The integration is carried out here over the surface $S(x')$ bounded by the loop Γ. Representing the plastic distortion u_{ik}^P connected with the dislocation in the form

$$u_{ik}^P = \int b_k \delta(x - x') dS_i = \int b_m \sigma_{mn,n}^k dS_i \tag{2}$$

we obtain for the elastic distortion u_{ik}, in accord with the Stokes theorem*,

$$u_{ik}(x) = u_{k,i} - u_{ik}^P = -\oint_\Gamma e_{ij\ell} \sigma_{\ell m}^k b_m dx_j' = -F_i^k, \tag{3}$$

i.e., the elastic distortion u_{ik} at the point x is equal to the i-th component of the generalized force F^{-k} exerted on the dislocation by the stress field produced by a single concentrated force applied at the point x in the direction $-k$. (We note that this rule holds for arbitrary sources of internal stresses.) Integration by parts transforms (2) into