ance is violated in the decays $K^0 \rightarrow e^+\nu\pi^-\pi^0$ and $\bar{K}^0 \rightarrow e^-\tilde{\nu}\pi^+\pi^0$. On the other hand, in the decays $K^+ \rightarrow e^+\nu\pi^+\pi^-$ and $K^- \rightarrow e^-\tilde{\nu}\pi^-\pi^+$ CP violation should lead to differences not only in the differential probabilities, but also in the partial widths of the decays (owing to "leakage" in the $K^+ \rightarrow e^+\nu 2\pi^0$ and $K^- \rightarrow e^-\tilde{\nu}2\pi^0$ channels). These effects will not be small if the $\Delta T = 1/2$ rule is violated in K_{e4} decay.

The author is grateful to I. Yu. Kobzarev for useful critical discussions and to V. I. Zakharov who called his attention to the role of the $\Delta T = 1/2$ rule in K_{e4} decays.

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*We assume that the rule $\Delta Q = \Delta S$ is valid.

**In this integration, expressions of the type $(p_\pi + p_\mu)^2$ are taken outside the integral sign, and the integration in the remaining expression reduces essentially to the substitution $p_\mu \rightarrow p_K - p_\nu$.

DISLOCATIONS IN AN ANISOTROPIC MEDIUM

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Submitted 11 August 1967
ZhETF Pis'ma 6, No. 8, 826-829 (15 October 1967)

Calculations of the elastic fields produced by dislocations in crystals serve as the basis for many applications of dislocation theory. However, allowance for the elastic anisotropy of the crystal can be made only in the case of straight-line dislocations, when the problem reduces to a planar one and admits the use of complex-variable methods (see [1]). The field of curvilinear dislocations is usually estimated very roughly in the elastic-isotropic approximation.

For a dislocation loop Γ with a Burgers vector b in an unbounded anisotropic medium, the displacement $u_k(x)$ coincides, in accord with the reciprocity theorem, with the work done by the stresses σ^k_{ij} produced by a unit force applied at the point x in the direction k on the discontinuities of the displacements b_j, needed to form the dislocation [2,3]

$$u_k(x) = \int b_i \sigma^k_{ij}(x^1 - x) dS_j. \tag{1}$$

The integration is carried out here over the surface $S(x')$ bounded by the loop Γ. Representing the plastic distortion u^P_{ik} connected with the dislocation in the form

$$u^P_{ik} = \int b_k \,\delta(x - x')\, dS_i = \int b_m \sigma^k_{mn,n}\, dS_i \tag{2}$$

we obtain for the elastic distortion u_{ik}, in accord with the Stokes theorem*,

$$u_{ik}(x) = u_{k,i} - u^P_{ik} = -\oint_\Gamma e_{ij\ell}\sigma^k_{\ell m} b_m dx'_j = -F^k_i, \tag{3}$$

i.e., the elastic distortion u_{ik} at the point x is equal to the i-th component of the generalized force F^{-k} exerted on the dislocation by the stress field produced by a single concentrated force applied at the point x in the direction -k. (We note that this rule holds for arbitrary sources of internal stresses.) Integration by parts transforms (2) into

$$u_{ik}(x) = \oint_{\Gamma} e_{nj\ell}(x_n - x'_n)\, \tau_j\, \sigma^k_{\ell m, i}\, ds, \tag{4}$$

where the vector $\tau_j = dx'/ds$ is directed along the tangent to the curve $\Gamma(s)$. Formulas (2) and (4) reduce the difficulty of finding the elastic dislocation field to the well known difficulties in constructing the Green's tensor for an isotropic medium [5]. We shall show that the sought-for field can be directly expressed in terms of the field of the elastic distortions of linear dislocations. According to (4) we have for a dislocation passing through the origin in the direction of the vector τ

$$u_{ik}(x, \tau) = e_{nj\ell}\, x_n\, \tau_j\, b_m \int_{-\infty}^{\infty} \sigma^k_{\ell m, i}(x - \tau s)\, ds. \tag{5}$$

Substituting $s = 1/s'$ in (5) and recognizing that

$$\sigma^k_{em, i}(a x) = \frac{\operatorname{sign} a}{a^3}\, \sigma^k_{em, i}(x),$$

we get

$$u_{ik}(x, \tau) = e_{nje}\, x_n\, \tau_j\, b_m \int_{-\infty}^{\infty} s' \operatorname{sign} s'\, \sigma^k_{em, i}(\tau - xs')\, ds'. \tag{6}$$

Applying the operator $D^2 = x_\beta x_\gamma(\partial^2 / \partial\tau_\beta \partial\tau_\gamma)$ to both sides of (6) and noting that

$$x_\beta \frac{\partial}{\partial \tau_\beta}\, \sigma^k_{em, i}(\tau - xs') = -\frac{d}{ds'}\, \sigma^k_{em, i}(\tau - xs'),$$

we get after integrating by parts

$$e_{nj\ell}\, x_n\, \tau_j\, \sigma^k_{\ell m, i}(\tau) = \frac{1}{2} D^2\, u_{ik}(x, \tau). \tag{7}$$

We replace in (7) x by τ and τ by $x - x'$. Then $D = \tau_i \partial/\partial x_i$ and formula (4) takes on the sought-for form

$$u_{ik}(x) = -\frac{1}{2} \oint_{\Gamma} \frac{\partial^2}{\partial x_m\, \partial x_n}\, \tau_m\, \tau_n\, u_{ik}(\tau, x - x')\, ds. \tag{8}$$

According to (8), the contribution made to the distortion by the linear dislocation segment between the points x_0 and $x_1 = x_0 + \tau s_0$ is

$$\begin{aligned} u_{ik}(x) &= \frac{1}{2}\, \tau_m \frac{\partial}{\partial x_m} \int_0^{s_0} \frac{d}{ds}\, u_{ik}(\tau, x - x_0 - \tau s)\, ds = \\ &= \frac{1}{2}(x_{1n} - x_{0n}) \frac{\partial}{\partial x_n} \left[u_{ik}(x_1 - x_0, x - x_1) - u_{ik}(x_1 - x_0, x - x_0) \right] \end{aligned} \tag{9}$$

To move one or two ends of the segment to infinity requires consideration of a transition to the limit in the form

$$\lim_{s \to \pm\infty} \tau_n \frac{\partial}{\partial x_n} u_{ik}(\tau, x - \tau s) = \lim_{s \to \pm\infty} s\tau_n \frac{\partial}{\partial x_n} u_{ik}(s\tau, x - \tau s) =$$

$$= \mp \lim_{s \to \pm\infty} x_n \frac{\partial}{\partial x_n} u_{ik}\left(x, \tau - \frac{x}{s}\right) = \pm u_{ik}(x, \tau).$$

We have allowed here for the fact that u_{ik} has an inhomogeneity index -1 as a function of the first argument and a zero inhomogeneity index as a function of the second argument. As a result, for a dislocation ray going off in the direction of τ to infinity, we have

$$u_{ik}(x) = \frac{1}{2}\left[u_{ik}(x, \tau) - \tau_n \frac{\partial}{\partial x_n} u_{ik}(\tau, x)\right]. \tag{10}$$

A superposition of fields of the type (8) or (9) makes it possible to construct the elastic field of any polygonal dislocation.

For a planar curvilinear loop, expression (8) can be simplified by considering the field in the plane of the loop only. In this case the operator $\tau_n \partial/\partial x_n$ is replaced by the operator $\tau r^{-1} \sin(\phi - \theta)\, \partial/\partial\theta$, where $r = |x - x'|$, ϕ is the azimuth of the vector τ, and θ is the azimuth of the vector $x - x'$. We get in lieu of (7) (after integrating by parts)

$$u_{ik}(x) = -\frac{1}{2}\oint \left(v_{ik} + \frac{\partial^2}{\partial\theta^2} v_{ik}\right)\frac{d\theta}{r}, \tag{11}$$

where

$$v_{ik}(\theta) = r \sin(\phi - \theta)\; u_{ik}(\tau, x - x') \tag{12}$$

is the orientational part (independent of the distance) of the distortion.

Using (8) - (11), we can write out expressions for the stress field and for the dislocation interaction and self-action forces, which determine the interaction energy and the self-energy of the dislocations. In particular, the use of (11) to calculate the self-action force of a flat loop yields directly Brown's result [6]. From formula (10) follows directly the law of ray interaction in dislocation nodes, making it possible to generalize the results of [7] to the case of arbitrary anisotropy (the law of interaction of the branches of a corner dislocation [8] turns out here to be a simple particular case). The moments of interaction of the dislocation rays can be investigated with the aid of polar diagrams that characterize the orientation dependence of the force (or energy) of interaction of parallel dislocations with equal Burgers vectors: the moments of interaction of the rays on these diagrams correspond to the distances between the curve and the tangents to it. The points of encounter of the tangents with the curve determine the position of the equilibrium rays. Equilibrium corner dislocations correspond to points with a common tangent. This result agrees with the criterion for the formation of corner points on the profiles of bodies with anisotropic surface energy [9] and explains why the results of investigations of corner points on dislocations [10], in which a seemingly invalid approximation of the linear energy was used, agree with experiment.

The foregoing examples give grounds for hoping that the developed theory will find many applications.

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* The result (2) is contained in implicit form in [4], which is devoted to the dynamics of dislocations.

SELF-MODULATION OF NONLINEAR PLANE WAVES IN DISPERSIVE MEDIA

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ZhETF Pis'ma 6, No. 8, 829 - 832 (15 October 1967)

In a nonlinear plane wave in which all the quantities depend on x and t only via the phase $\theta = kx - \omega t$ (we shall call such a wave stationary), the frequency ω is determined not only by the wave number k, but also by other parameters which are assumed small in the linear theory. We consider here a case when only the amplitude is such a parameter, i.e., we assume that the nonlinear dispersion equation is of the form

$$\omega = \omega(k^2, a^2) \tag{1}$$

(the medium is assumed for simplicity to be isotropic and the wave is assumed linearly polarized). The properties of the stationary waves, and particularly relation (1), are usually obtained in relatively simple fashion from the general equations describing the given wave field.

We consider in this note the evolution of the local perturbations of nonlinear stationary waves. It is assumed here that the spatial scale of the perturbation is large compared with the wavelength, so that the wave can be regarded as quasistationary. Thus, the amplitude $a(x,t)$ of the wave is a slowly varying function, and the phase takes the form $\theta = k_0x - \omega_0t + \phi(x,t)$, where k_0 and ω_0 are the "unperturbed" wave number and frequency, which satisfy Eq. (1) with $a = a_0$ (a_0 is the unperturbed amplitude), and ϕ_x/k_0 and ϕ_t/ω_0 are small quantities. If the amplitude a is also regarded as a small quantity, then the system of equations for a and ϕ, accurate to terms of order a^2 and $(\phi_x/k_0)^2$ inclusive, takes the form

$$\begin{gathered} \phi_\tau + \frac{1}{2}\phi_\xi^2 - \mu(a^2 - a_0^2) - \frac{1}{2a}a_{\xi\xi} = 0, \\ (a^2)_\tau + (a^2\phi_\xi)_\xi = 0, \end{gathered} \tag{2}$$

where

$$\begin{gathered} \xi = x - u_0 t, \quad \tau = tu_0', \quad \mu = -\frac{1}{u_0'}\left(\frac{\partial\omega}{\partial a^2}\right)_{a=0,\, k=k_0}, \\ u_0 = \frac{\partial\omega(k_0, 0)}{\partial k_0}, \quad u_0' = \frac{\partial^2\omega(k_0, 0)}{\partial k_0^2} \end{gathered} \tag{3}$$