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* The result (2) is contained in implicit form in [4], which is devoted to the dynamics of dislocations.

SELF-MODULATION OF NONLINEAR PLANE WAVES IN DISPERSIVE MEDIA

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In a nonlinear plane wave in which all the quantities depend on x and t only via the phase $\theta = kx - \omega t$ (we shall call such a wave stationary), the frequency ω is determined not only by the wave number k, but also by other parameters which are assumed small in the linear theory. We consider here a case when only the amplitude is such a parameter, i.e., we assume that the nonlinear dispersion equation is of the form

$$\omega = \omega(k^2, a^2) \quad (1)$$

(the medium is assumed for simplicity to be isotropic and the wave is assumed linearly polarized). The properties of the stationary waves, and particularly relation (1), are usually obtained in relatively simple fashion from the general equations describing the given wave field.

We consider in this note the evolution of the local perturbations of nonlinear stationary waves. It is assumed here that the spatial scale of the perturbation is large compared with the wavelength, so that the wave can be regarded as quasistationary. Thus, the amplitude a(x,t) of the wave is a slowly varying function, and the phase takes the form $\theta = k_0x - \omega_0t + \phi(x,t)$, where k_0 and ω_0 are the "unperturbed" wave number and frequency, which satisfy Eq. (1) with $a = a_0$ (a_0 is the unperturbed amplitude), and ϕ_x/k_0 and ϕ_t/ω_0 are small quantities. If the amplitude a is also regarded as a small quantity, then the system of equations for a and ϕ, accurate to terms of order a^2 and $(\phi_x/k_0)^2$ inclusive, takes the form

$$\phi_\tau + \frac{1}{2}\phi_\xi^2 - \mu(a^2 - a_0^2) - \frac{1}{2a}a_{\xi\xi} = 0,$$
$$(a^2)_\tau + (a^2\phi_\xi)_\xi = 0, \quad (2)$$

where

$$\xi = x - u_0t, \quad \tau = tu_0', \quad \mu = -\frac{1}{u_0'}\left(\frac{\partial\omega}{\partial a^2}\right)_{a=0,\, k=k_0},$$
$$u_0 = \frac{\partial\omega(k_0,0)}{\partial k_0}, \quad u_0' = \frac{\partial^2\omega(k_0,0)}{\partial k_0^2} \quad (3)$$

(ξ is the coordinate in a frame moving at a group velocity u_0 relative to the medium). If we assume for (1) the equation $\omega^2 = c^2k^2/(\epsilon_0 + \epsilon'|E|^2)$, then we obtain the nonlinear-optics equations for weakly-modulated waves [1,2].

For small perturbations ($a - a_0 \ll a_0$, $\varphi \ll 1$), Eq. (2) leads to a dispersion equation in the form

$$\Omega = \frac{\kappa}{2}(\kappa^2 - \kappa_0^2)^{1/2}, \quad \kappa_0^2 = 4\mu a_0^2, \tag{4}$$

where κ is the wave number of the perturbation. Thus, when $\mu > 0$ (this is just the case considered henceforth) the perturbations with $\kappa < \kappa_0$ grow exponentially, meaning a modulational instability of the stationary wave [3-5].

Let the initial perturbation be determined by the conditions

$$\phi(\xi, 0) = 0, \quad a(\xi, 0) = a_0[1 + f(\xi/\ell)], \tag{5}$$

where $f(z)$ is an even function that vanishes when $z \to \infty$. Then the asymptotic form of the perturbation during the linear stage of the process at $\xi \ll \kappa_0\tau$ will be as follows:*

$$a = a_0 + \text{const} \exp(\kappa_0^2 \tau/4) \cos(\kappa_0 \xi/\sqrt{2}). \tag{6}$$

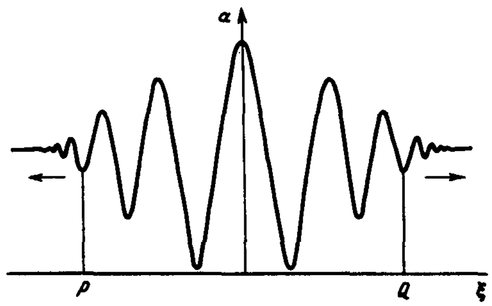
Expression (6) describes an envelope wave which is stationary in the reference frame under consideration (i.e., propagating with a group velocity u_0 relative to the medium). In the other limiting case $\xi \gg \kappa_0\tau$, we obtain expressions of the form $a = a_0 + a_- + a_+$, where $a_\pm$ denotes envelope waves diverging from the center and having a wave number $\kappa(\xi, \tau) \gg \kappa_0$. Their local frequency $\Omega(\xi, \tau)$ is determined in terms of κ by Eq. (4), the group velocities $d\Omega/d\kappa$ are equal to ξ/τ, and the amplitudes attenuate when $\xi \to \pm\infty$ (we shall not present here the complicated expressions for $a_\pm$).

In the linear stage of the process, an important role is played by solitary envelope waves (solitons), described by the relations

$$a = A\,\text{sech}(A\sqrt{\mu}\xi), \qquad \phi = \frac{1}{2}\mu(A^2 - 2a_0^2)\tau, \tag{7}$$

which, as is well known [5-7], are exact solutions of the equations in (2).

A qualitative analysis of the evolution at sufficiently large τ, based on the hypothesis that solitons (7) are stable, leads to the following picture (see the figure). The oscillations of the amplitude in the central region, which had in the linear stage the form (6), are transformed - with increasing depth of modulation - into the solitons (7). With increasing distance from the center ($\xi = 0$), the variation of the amplitude and of the phase can be asymptotically represented in the form



$$a = A\,\text{dn}(A\sqrt{\mu}\xi, s), \quad \phi = \frac{1}{2}\mu\int(A^2 + B^2 - 2a_0^2)d\tau, \tag{8}$$
$$s^2 = (A^2 - B^2)/A^2,$$

where dn(z, s) is the Jacobi elliptic function, s is its modulus, and the quantities A and B (A = max a, B = min a) are slowly varying functions of ξ and τ (compared with the modulation lengths). (When A and B are constant, expressions (8) are, like (7), exact solutions of the equations in (2).) When $\xi \to 0$, B vanishes and (8) goes over into (7). The region of applicability of expressions (8) terminates near the boundaries of the "deeply modulated" region PQ (see the figure). Outside this region, the oscillations of the amplitude are diverging waves similar to those produced during the linear stage of the process when $\xi \gg \kappa_0\tau$. With increasing distance from the points P and Q, the group velocities of the diverging waves take on the asymptotic form ξ/τ. It can also be shown that the width of the region PQ increases more rapidly than τ (as $\tau \to \infty$); the number of solitons formed in the central region PQ is then accordingly increased.

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* The function f(z) can always be chosen such that (6) holds for sufficiently large τ and ξ.

RADIOACTIVE NUCLEI IN SOLAR COSMIC RAYS

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We wish to call attention in this note to the possible existence of an interesting phenomenon in solar cosmic rays.

According to existing notions (see [1,2]), the cosmic rays are generated on the sun during the time of chromospheric flares, and prior to emerging to the interplanetary medium a particle traverses in the solar atmosphere a path $\ell = 10^9$ cm with an average hydrogen concentration $n_H = 3 \times 10^{13}$ cm^{-3}. The flux of radioactive nuclei of any type i in the energy interval (E_1', E_2'), produced as a result of collision between the accelerated particles and the hydrogen of the solar atmosphere, can be readily calculated from the formula

$$F_i(E_1', E_2') = n_H l \int_{E_1}^{E_2} F_j(E)\sigma(E)dE = n_H \ell \overline{\sigma(E)} F_j(E_1, E_2), \quad (1)$$

where $\sigma(E)$ is the cross section of this reaction, and $F_j(E_1, E_2)$ is the flux of nuclei of type j in the energy interval (E_1, E_2). We have used here the fact that the path traversed by the particle in the solar atmosphere is equal to the thickness of the region of the flare, regardless of the particle energy [2].

We shall not consider here the production of all possible radioactive nuclei, and in-