photocurrent J_{np}. For $m = 0.01m_0$, $E_g = 0.23$ eV, $\lambda = 10$ μ, and $E_0 = 10^5$ V/cm we obtain $J_{np} = -0.3J_0$ ($\theta = 0$).

This effect can be separated from the photoconductivity connected with electron "heating." The increment of the average electron energy in the field of the wave is determined by the mechanisms of light absorption by the electron and subsequent transfer of energy to the lattice. If both processes occur as a result of an interaction between the electron and optical vibrations, then we obtain from the energy-balance equation the rough estimate $\Delta\epsilon \sim e^2E_0^2/m\omega^2$. Since τ_p depends on the electron energy, we get an additional dependence of the current J_{np} on the light intensity. This dependence can be neglected if $\Delta\epsilon < \epsilon_0$ (ϵ_0 is the Fermi energy or $3kT/2$). In the foregoing numerical example, this inequality is satisfied for the degenerate case when $\Delta\epsilon \sim 0.2$ eV if $E_0 \lesssim 10^5$ V/cm and for $\Delta\epsilon \sim 0.02$ eV if $E_0 \lesssim 10^4$ V/cm. The mobility $e\tau_p/m$ depends on the electron energy via the relaxation time and the mass. Owing to the dependence of τ_p on $\Delta\epsilon$, a photocurrent proportional to $\Delta\epsilon/\epsilon_0$ is produced, and the dependence of m on $\Delta\epsilon$ leads to a photocurrent proportional to $\Delta\epsilon/E_g$ (in (1) T is the temperature of electron-gas "heating"). The energy increment $\Delta\epsilon$ does not depend on θ, since the electrons are heated only by the light (the field E_c is weak). Therefore, the photocurrent due to "heating" drops out of the difference between the photocurrents at $\theta = 0$ and $\theta = \pi/2$. We note that in principle, another method of eliminating the "heating" effect is possible, namely the use of very short light pulses of duration shorter than the electron heating time.

Owing to the nonlinear dependence of the hole velocity on the momentum [3], an effect similar to that considered above should be observed in p-type germanium and silicon.

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CONTRIBUTION TO THE THEORY OF THE INTERMEDIATE STATE OF CURRENT-CARRYING SUPERCONDUCTORS

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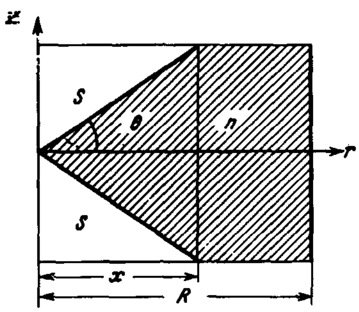

Fig. 1

As shown by London [1], a periodic structure (intermediate state) is produced in a superconductor under the influence of an electric current exceeding the critical value $J_{c0} = cH_cR/2$ (H_c is the critical magnetic field and R is the radius of the sample); one period of this structure is shown in Fig. 1. The macroscopic description used in [1], corresponding to the condition $\theta \to 0$, makes it possible to calculate the resistance of the sample, but not the angle θ. The accuracy of the macroscopic description is thus left undetermined. In this paper we calculate the angle θ and obtain the corrections to the critical current and to the resistance necessitated by the fact that θ is finite.*

If we introduce a cylindrical coordinate system (r, φ, z) such as shown in Fig. 1, then it is clear from symmetry considerations that the only nonvanishing magnetic-field component is $H_\varphi(r, z) \equiv H$. In the normal phase, it satisfies the relation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) - \frac{H}{r^2} + \frac{\partial^2 H}{\partial z^2} = 0. \quad (1)$$

In the region $r \le x$, $|z| \le r\theta$ it is convenient to seek the solution in the form of an expansion in the small ratio z^2/r^2:

$$H = f_0(r) + \frac{z^2}{2r^2} f_2(r).$$

From (1) we then get $f_2 = f_0 - r\,\partial/\partial r(r\,\partial f_0/\partial r)$. On the interface between the normal and superconducting phases, i.e., at $z = \pm\theta r$, the value of H should be H_c, or

$$f_0 + \frac{\theta^2}{2}\left\{f_0 - r\frac{\partial}{\partial r}\left(r\frac{\partial f_0}{\partial r}\right)\right\} = H_c$$

At small values θ, the solution of the last equation is $f_0 = H_c(1 - \theta^2/2)$, and thus

$$H = H_c\left(1 - \frac{\theta^2}{2} + \frac{z^2}{2r^2}\right), \quad (2)$$

In the region $r > x$ we get from (1) for the field $\overline{H}$ averaged over z the equation $\partial/\partial r(r\,\partial\overline{H}/\partial r) - \overline{H}/r = 0$, whence $\overline{H} = \alpha/r + \beta r$ (α and β are constants). The condition for the continuity of $\overline{H}$ at $r = x$ yields

$$\frac{\alpha}{x} + \beta x = H_c\left(1 - \frac{\theta^2}{3}\right). \quad (3)$$

We have taken into account the fact that $\overline{H} = (H_c r/x)(1 - \theta^2/3)$ when $r < x$, as can be readily seen from (2) and from the fact that $H = 0$ in the superconducting phase.

The difference $H - \overline{H}$ decreases rapidly with increasing r in the region $r > x$, and since the inequality $R - x \gg \theta x$ is always satisfied (this is seen from the result), we can assume $H = \overline{H}$ practically everywhere if $r > x$.

The magnetic field on the surface of the sample should equal $2j/cR$ (j is the total current). From this we get

$$\frac{\alpha}{R} + \beta R = 2j/cR. \quad (4)$$

if $\theta = 0$, then it follows from (3) and (4) that $\alpha = \alpha_0 \equiv H_c x_0/2$, $\beta = \beta_0 \equiv H_c/2x_0$, and $x_0/R = j/j_{c0} - \sqrt{j^2/j_{c0}^2 - 1}$. At small θ, putting $\alpha = \alpha_0 + \delta\alpha$, $\beta = \beta_0 + \delta\beta$, and $x = x_0 + \delta x$, and linearizing equations (3) and (4) relative to the quantities $\delta\alpha$, $\delta\beta$, and δx, we get

$$\delta a = -\frac{R^2 x_0}{R^2 - x_0^2} H_c \frac{\theta^2}{3}, \quad \delta\beta = \frac{x_0}{R^2 - x_0^2} H_c \frac{\theta^2}{3}. \tag{5}$$

For a given value of the current j, the thermodynamic potential, the density of which is $\tilde{\Phi} = \Phi - HB/4\pi$ [3], should tend to its lowest possible value.** If we reckon Φ from the value corresponding to the superconducting phase ($\Phi_s = 0$), then in the normal phase we have $\Phi_n = (H_c^2 + H^2)/8\pi$.

The total thermodynamic potential is equal to the sum $\tilde{\Phi}_1 + \tilde{\Phi}_2 + \tilde{\Phi}_3$, where

$$\tilde{\Phi}_1 \equiv \int_x^R 2\pi r\,dr \frac{1}{8\pi}\{H_c^2 - (\frac{a}{r} + \beta r)^2\} = \frac{H_c^2}{8}(R^2 - x^2) - \frac{1}{4}\{a^2 \ln\frac{R}{x} +$$

$$+ a\beta(R^2 - x^2) + \frac{\beta^2}{4}(R^4 - x^4)\}, \tag{6}$$

$$\tilde{\Phi}_2 \equiv \frac{1}{2x\theta}\int_0^x \frac{r\,dr}{4}\int_{-r\theta}^{r\theta} dz\{H_c^2 - H_c^2(1 - \theta^2 + \frac{z^2}{r^2})\} = \frac{H_c^2}{18}\theta^2 x^2; \tag{7}$$

$$\tilde{\Phi}_3 = \frac{H_c^2}{8\pi}\Delta 2\pi x^2 \frac{1}{2x\theta} = \frac{H_c^2}{8}\frac{x\Delta}{\theta}. \tag{8}$$

The last term is connected with the surface tension on the interface, the value of which is written, as usual in the form $H_c^2\Delta/8\pi$. The constant, which is equal to the potential of the field outside the sample, is omitted throughout.

Using the inequalities (5), we can readily separate from $\tilde{\Phi}_1$ the part independent of θ:

$$\tilde{\Phi}_1 = \tilde{\Phi}^{(0)} + \frac{H_c^2 R^2}{12}\theta^2\{\frac{x_0^2/R^2}{1 - x_0^2/R^2}\ln\frac{R}{x_0} + \frac{1}{4} - \frac{3}{4}\frac{x_0^2}{R^2}\}, \tag{9}$$

where $\tilde{\Phi}^{(0)}$ does not depend on θ. When $(j - j_{c0})/j_{c0} \ll 1$ we have

$$\tilde{\Phi}^{(0)} = -\frac{H_c^2 R^2}{12}(2\frac{J - J_{c0}}{J_{c0}})^{3/2}. \tag{10}$$

By minimizing the sum of the expressions (7), (8), and (9) we get the equilibirium value of θ:

$$\theta = [\frac{\Delta}{R\Psi(j/j_c)}]^{1/3}, \tag{11}$$

where

$$\Psi(j/j_c) = \frac{4R}{3x_0}\{\frac{x_0^2/R^2}{1 - x_0^2/R^2}\ln\frac{R}{x_0} + \frac{1}{4} - \frac{3}{4}\frac{x_0^2}{R^2}\} + \frac{8}{9}\frac{x_0}{R}.$$

When j is close to j_{c0} we have $\theta = (9\Delta/8R)^{1/3}$. At larger currents ($j \gg j_{c0}$) we have $\theta = (3\Delta j_{c0}/2RJ)^{1/3}$.

If the entire sample were in the superconducting state, then, as is clear from the foregoing, the thermodynamic potential would be equal to zero. Therefore the critical current is determined from the condition for the vanishing of the sum of expressions (7), (8), and (9). Using (10), we find the correction to j_{c0} due to the finite nature of θ:

$$\frac{j_c - j_{co}}{j_{co}} = \frac{1}{2}\left(\frac{9}{2}\right)^{2/3}\left(\frac{\Delta}{3R}\right)^{4/9}. \qquad (12)$$

The correction to the resistance W can be readily determined by noting that the constant β is connected with the z-component of the electric field by the relation $\beta = 2\pi\sigma E_z/c$ (σ is the conductivity of the normal phase). We have:

$$\frac{W}{W_n} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{j_{co}^2}{j^2}}\right) + \frac{j_{co}}{j}\,\frac{x_0 R}{R^2 - x_0^2}\,\frac{\theta^2}{3}, \qquad (13)$$

where W_n is the resistance in the normal state. We write one more expression for the resistance W_c when $j - j_c$. From (12) and (13) we get

$$\frac{W_c}{W_n} = \frac{1}{2} + \frac{1}{2}\left(\frac{9}{2}\right)^{1/3}\left(\frac{\Delta}{3R}\right)^{2/9}. \qquad (14)$$

Figure 2 shows a comparison of $W_c(R)$ given by formula (14) with Meissner's experimental data [4] for tin. In the calculations we substituted $\Delta \approx 1 \times 10^{-4}$ cm, corresponding (see [5]) to the average temperature $T \approx 3.5°$ used in [4]. The agreement between theory and experiment turned out to be even better than expected.

We note also that the corrections to the magnitude of the critical current are much smaller than those to W_c. This fact also agrees with experiment.

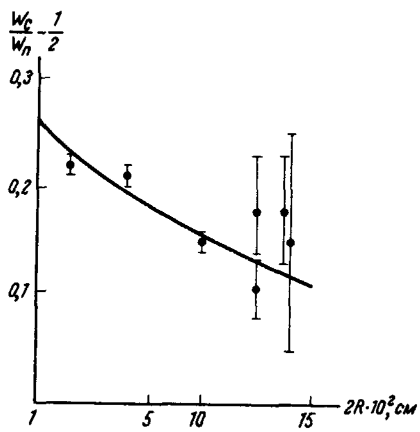

Fig. 2

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* Kuper reached the conclusion that $\theta \sim 1$ and therefore macroscopic description is essentially impossible. We shall see, however, that this conclusion is in error, for actually $\theta \ll 1$.

** It is assumed that the conductivity σ is not too small, so that the deviation from equilibrium due to the release of Joule heat can be neglected.