then this will be a direct refutation of the geodesic principle.

Before sufficiently exact verification data are obtained, we believe that the doubts concerning the metric nature of the gravitational field are unfounded.

[1] R. Dicke and H. Mark Goldenberg, Phys. Rev. Lett. 18, 313 (1967).
[2] N. M. Polievktov-Nikoladze, ZhETF Pis. Red. 2, 551 (1965) [JETP Lett. 2, 342 (1965)].
[3] N. M. Polievktov-Nikoladze, Zh. Eksp. Teor. Fiz. 52, 1360 (1967) [Sov. Phys.-JETP 25, 904 (1967)].
[4] L. D. Landau and E. M. Lifshitz, Classical Theory of Fields, Addison-Wesley, 1962.

INTERBAND TRANSITIONS IN SEMICONDUCTORS IN A MAGNETIC FIELD UNDER THE INFLUENCE OF A STRONG ELECTROMAGNETIC WAVE

V. Zh. Zhukovskii and A. A. Sokolov
Physics Department, Moscow State University
Submitted 6 July 1967
ZhETF Pis'ma 6, No. 9, 876-879 (1 November 1967)

It is shown in a number of papers [1-2] that intense laser emission is capable of causing multiquantum transitions of electrons from the valence band of a conductor to the conduction band. In this note we wish to show that application of a constant magnetic field makes it possible to obtain a resonant increase of the absorption coefficient in the presence of a strong electromagnetic wave. The transitions produced thereby are forbidden by the selection rules with respect to the magnetic quantum number and have a probability proportional to certain powers of the intensity of the external radiation.

The potentials of the constant magnetic field parallel to the z axis and of a plane circularly-polarized wave are specified in the form

$$A(H) = (0,\ xH,\ 0),\quad A(t) = (b\cos\omega t,\ b\sin\omega t,\ 0).$$

For an electron in vacuo, the solution of the relativistic wave equations in such field is known. To find the wave functions of an electron in a semiconductor, we introduce the creation and annihilation operators a^+ and a:

$$a = \frac{1}{r_0}\left(\frac{cp_x}{eH} - i(x - x_0)\right),\quad [a, a^+] = 1,\quad x_0 = -\frac{cp_y}{eH},\quad r_0 = \left(\frac{2c\hbar}{eH}\right)^{1/2}. \tag{1}$$

We use them to define the operator

$$\Lambda = \exp\left[-\frac{i}{\hbar}\int^{t}\gamma(t')\,dt' + \mu a^+ - \mu^* a\right],$$

where $\mu(t)$ and $\gamma(t)$ satisfy the equations

$$\dot{\mu} = -i\omega_c\mu - i\frac{eb}{mcr_0}e^{-i\omega t},$$
$$\gamma = -\frac{im\omega_c r_0^2}{4}(\dot{\mu}\mu^* - \mu\dot{\mu}^*) + \frac{1}{2}mr_0^2|\dot{\mu}|^2,\quad \omega_c = \frac{eH}{mc}. \tag{2}$$

Let us assume that the conduction band and the valence band are parabolic and nondegenerate. The wave functions can be written in such a case as follows:

$$\Psi = U_{i0}(r) f_i(r, t) \tag{3}$$

$f_1 = \Lambda_1 \overline{\Phi}_n \exp(-i\epsilon_1 t/\hbar)$ - for valence band 1,

$f_2 = \Lambda_2 \Phi_{n'} \exp(-i\epsilon_2 t/\hbar)$ - for conduction band 2,

where U_{i0} is the periodic part of the Bloch function for k = 0 and $\epsilon_1 = -E_n - \epsilon_1^0$ and $\epsilon_2 = E_{n'} + \epsilon_2^0$ are the values of the energy in bonds 1 and 2 in a magnetic field; $\epsilon_1^0 + \epsilon_2^0 = \epsilon_g$ is the width of the forbidden band,

$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right) + (\hbar k_z)^2/2m,$$

and $\overline{\Phi}_n(r) \exp(-iE_n t/\hbar)$ is the eigenfunction of the electron in a magnetic field.

If the electron is at a level n in the absence of the electromagnetic field, then a strong alternating field with photon energy $\hbar\omega < \epsilon_g$ makes possible transitions inside the band, as a result of which the following distribution is established for the probability of finding the electron at the levels ℓ:

$$W_l = \begin{cases} I^2_{\ell,n}(\rho), & \ell \geq n \\ I^2_{n,\ell}(\rho), & \ell < n, \end{cases} \tag{4}$$

where

$$I_{\ell,n}(\rho) = (\ell! n!)^{-1/2} e^{-\rho/2} \rho^{(\ell-n)/2} Q_n^{\ell-n}(\rho);$$

$Q_n^{\ell-n}$ is a Laguerre polynomial and $\rho = e^2b^2/[mcr_0(\omega - \omega_c)]^2$. We assume here that the frequency ω differs from cyclotron frequency and therefore only nonresonant transitions are possible inside the band.

The matrix element of the transition from the valence band to the conduction band under the influence of the field $E = \epsilon E_\sim \exp(-i\omega' t)$ is

$$M_{12} = \frac{eE_\sim}{m\omega'}(p_{12}\vec{\epsilon}) I_{n',n}(\xi); \quad \xi = \frac{e^2F^2}{[m^* r_0(\omega + \omega_{C1})(\omega - \omega_{C2})]^2}, \tag{5}$$

where p_{1q} is the matrix element of the dipole transition at k = 0, and $F = b\omega/c$ is the amplitude of the wave field; ω_{C1} and ω_{C2} are the cyclotron frequencies in bands 1 and 2, and m^* is the reduced mass. In such a transition, the selection rules are satisfied for the wave numbers $k_{z1} = k_{z2}$, $k_{y1} = k_{y2}$, but not for the quantum numbers n', since transitions to the states $n' \neq n$ are possible when $\xi \neq 0$. We note that in constant crossed fields there are likewise no selection rules with respect to n [5].

The frequency for the transition (5) should satisfy the condition

$$\hbar\omega' = (n - n')\hbar\omega + \hbar\omega_{C1}\left(n + \frac{1}{2}\right) + \hbar\omega_{C2}\left(n' + \frac{1}{2}\right) + \frac{(\hbar k_z)^2}{2m^*} + \epsilon_g + \frac{e^2F^2}{2m^*(\omega + \omega_{C1})(\omega - \omega_{C2})}. \tag{6}$$

The last term on the right side of this equation corresponds to a constant shift of the absorption edge, and the sign of the shift depends on the ratio of ω to ω_{C2}. In addition, when $n' \neq n$ simultaneous absorption of several photons $\hbar\omega$ from an intense wave becomes possible, and the matrix element of the transition is proportional to $\xi^{n'-n}$.

Using (5), we can find the absorption coefficient:

$$\alpha = \frac{(p_{12}\vec{\epsilon})^2 e^3 H (2m^*)^{1/2}}{n_0 c^2 \hbar^2 m\omega'} \sum_{n',n} I^2_{n',n}(\xi)\Big[\hbar\omega' + (n'-n)\hbar\omega - \hbar\omega_1\left(n+\frac{1}{2}\right) - \hbar\omega_{C2}\left(n'+\frac{1}{2}\right) - \epsilon_g - \frac{e^2F^2}{2m^*(\omega+\omega_{C1})(\omega-\omega_{C2})}\Big]^{-1/2} \qquad (7)$$

The maximum absorption will be observed for resonant values of the frequencies from (6) at $k_z = 0$, but with this α will nevertheless be finite, owing to the presence of damping. When $n' - n$ is large, the function $I_{n',n}$ behaves like the Bessel function $J_{n'-n}(2\sqrt{n\xi})$, and consequently oscillations of the absorption coefficient can be observed, depending on the value of ξ.

In a strong field with $\xi \gg 1$, for transitions between small quantum numbers, the matrix element proportional to $e^{-\xi/2}$ will yield an exponential damping of the transition probability.

When $\omega \ll \omega_C$ the electrons move in an almost constant field and therefore formulas (5), (6), and (7) go over into the corresponding formulas obtained by Aronov [5] for constant crossed fields. The parameter ξ for intense microwave or infrared radiation can become close to unity ($\xi \sim 1$). To this end it is necessary to specify, for example, the values $m^* = 10^{-28}$ g, $\omega = 10^{13}$ sec, $H = 10^4$ G, and an electric field intensity $F = 10^4$ V/cm, which are perfectly attainable at the present time.

[1] R. Braunstein, Phys. Rev. 125, 475 (1962).
[2] L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1945 (1964) [Sov. Phys.-JETP 20, 1307 (1965)].
[3] M. Inoue and Y. Toyozawa, J. Phys. Soc. Japan 20, 363 (1965).
[4] P. J. Redmond, J. Math. Phys. 6, 1163 (1964).
[5] A. G. Aronov, Fiz. Tverd. Tela 5, 552 (1963) [Sov. Phys.-Solid State 5, 402 (1963)].

INFLUENCE OF QUANTUM FLUCTUATIONS ON THE WIDTH OF THE BAND OF RADIATED FREQUENCIES IN A SUPERCONDUCTING TUNNEL STRUCTURE

Yu. M. Ivanchenko
Donets Physico-technical Institute, Ukrainian Academy of Sciences

Submitted 3 September 1967
ZhETF Pis'ma 6, No. 9, 879-881 (1 November 1967)

In an earlier paper [1], the author considered the Josephson effect [2] with allowance for the quantum character of the interaction between the electrons and the electromagnetic field. It was shown there that quantization leads to the appearance of a radiated-frequency band having a finite width. However, the width was not calculated in [1], and was connected with the noise introduced from the external circuit. In fact, the width arises in natural fashion as a result of the quantum fluctuations of the charge, and dissipative noise leads