The last term on the right side of this equation corresponds to a constant shift of the absorption edge, and the sign of the shift depends on the ratio of ω to ω_{C2}. In addition, when $n' \neq n$ simultaneous absorption of several photons $\hbar\omega$ from an intense wave becomes possible, and the matrix element of the transition is proportional to $\xi^{n'-n}$.

Using (5), we can find the absorption coefficient:

$$\alpha = \frac{(p_{12}\vec{\mathcal{E}})^2 e^3 H (2m^*)^{1/2}}{n_0 c^2 \hbar^2 m\omega'} \sum_{n',n} I^2_{n',n}(\xi)\left[\hbar\omega' + (n'-n)\hbar\omega - \right.$$
$$\left. - \hbar\omega_1\left(n+\frac{1}{2}\right) - \hbar\omega_{C2}\left(n'+\frac{1}{2}\right) - \varepsilon_g - \frac{e^2F^2}{2m^*(\omega+\omega_{C1})(\omega-\omega_{C2})}\right]^{-1/2} \tag{7}$$

The maximum absorption will be observed for resonant values of the frequencies from (6) at $k_z = 0$, but with this α will nevertheless be finite, owing to the presence of damping. When $n' - n$ is large, the function $I_{n',n}$ behaves like the Bessel function $J_{n'-n}(2\sqrt{n\xi})$, and consequently oscillations of the absorption coefficient can be observed, depending on the value of ξ.

In a strong field with $\xi \gg 1$, for transitions between small quantum numbers, the matrix element proportional to $e^{-\xi/2}$ will yield an exponential damping of the transition probability.

When $\omega \ll \omega_C$ the electrons move in an almost constant field and therefore formulas (5), (6), and (7) go over into the corresponding formulas obtained by Aronov [5] for constant crossed fields. The parameter ξ for intense microwave or infrared radiation can become close to unity ($\xi \sim 1$). To this end it is necessary to specify, for example, the values $m^* = 10^{-28}$ g, $\omega = 10^{13}$ sec, $H = 10^4$ G, and an electric field intensity $F = 10^4$ V/cm, which are perfectly attainable at the present time.

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INFLUENCE OF QUANTUM FLUCTUATIONS ON THE WIDTH OF THE BAND OF RADIATED FREQUENCIES IN A SUPERCONDUCTING TUNNEL STRUCTURE

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In an earlier paper [1], the author considered the Josephson effect [2] with allowance for the quantum character of the interaction between the electrons and the electromagnetic field. It was shown there that quantization leads to the appearance of a radiated-frequency band having a finite width. However, the width was not calculated in [1], and was connected with the noise introduced from the external circuit. In fact, the width arises in natural fashion as a result of the quantum fluctuations of the charge, and dissipative noise leads

to additional broadening and, in the case of high intensity to annihilation of the Josephson current. In this paper we calculate the emitted-frequency band width connected with the quantum fluctuations of the charge.

For the homogeneous case we can easily obtain from the Hamiltonian obtained in [1], neglecting retardation [2], accurate to terms of first order in the transparency of the barrier, a Hamiltonian that takes into account the discrete nature of the surface charge. A Hamiltonian of this type was considered by Anderson in [4].

Calculating the current I accurate to terms $\sim\delta^{-2} = 2e^3/\hbar I_0 C$ (I_0 is the maximum Josephson current [5], C the capacitance of the tunnel contact), we get

$$I = I_0 \sqrt{\frac{2}{\pi\delta}} \sum_n \exp\left[-\frac{2}{\delta}\left(n-\frac{1}{4}\right)^2\right] \sin\left[\omega_0 + \left(n+\frac{1}{2}\right)\omega_C\right]t, \tag{1}$$

where $\omega_0 = 2ev/\hbar$ and $\omega_c = 4e^2/c\hbar$.

From Eq. (1) we obtain an expression for the emitted-frequency band width connected with the quantum fluctuations of the charge of the tunnel-junction capacitance

$$\Delta\omega = \frac{4e^2}{c\hbar}\left(\frac{\hbar I_0 C}{2e^3}\right)^{1/4}. \tag{2}$$

Relations (1) and (2) hold true for $e^2/c \ll ev \ll \Delta$ in the absence of an external magnetic field and under the condition that the dispersion of the surface charge, which is connected with dissipative processes, is much smaller than the dispersion resulting from the quantum fluctuations. This can lead to an additional condition, which can be readily obtained by using the Nyquist theorem,

$$kT < \left(\frac{\hbar I_0 e}{2C}\right)^{1/2} = kT_c. \tag{3}$$

If the inequality (3) is satisfied, then the main contribution to the emission line width will be made by quantum fluctuations.

Let us compare now the obtained results with the experimental data. Junctions with $C \sim 0.3 \times 10^4$ cm and $\hbar I_0/re \sim 3 \times 10^{-11}$ erg were used in [6]. From (3) we get $T_c = 4°K$. Inasmuch as the experiment was performed at 1.4°K, the quantum fluctuations will make the main contribution to the width. Relation (2) yields $\Delta f = \Delta\omega/2\pi \sim 3.5 \times 10^7$ Hz. In order of magnitude Δf coincides with that measured in [6]. However, the additional structure noted in [6], with maxima spaced $\sim 2 \times 10^7$ Hz apart, cannot be explained within the framework of this model.

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CONCERNING THE ANOMALIES OF THE ELECTRONIC PROPERTIES OF RARE-EARTH METALS

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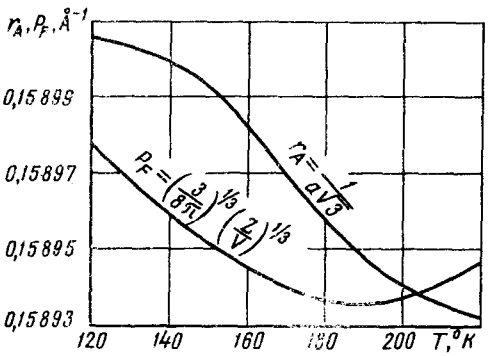
Studies of the crystal structure of rare-earth metals (gadolinium [1], terbium [2], and dysprosium [3]) in the magnetically ordered state, have revealed maxima on the plots of the atomic volume against the temperature (at $T_{max} \sim 200°K$ for Gd, 210°K for Tb, and 145°K for Dy). These maxima are connected with the different temperature dependences of the periods of the hexagonal close-packed lattices ($da/dT > 0$, $dc/dT < 0$) in the ferromagnetic region (Gd, Tb) or in the antiferromagnetic region (Dy). At approximately the same temperatures, anomalies were observed in a number of physical properties [4]: the specific magnetization, the electric resistivity, the thermal emf, etc. The purpose of the present note is to call attention to the fact that the anomalies of the physical properties of the rare-earth metals at T_{max} may be the consequence of a change in the topology of the Fermi surface [5,6], resulting from the intersection of this surface with the boundaries of the first Brillouin zone. In this case, the change in the dimensions of the Brillouin zone is due not to application of pressure (as treated in [5]), but to anomalies in the thermal deformation of the lattice.

If we confine ourselves for similicity to an isotropic dispersion law, then it is easy to verify that when the atomic volume ($V = a^2 c\sqrt{3}/H$) has a nonmonotonic temperature dependence, the situation illustrated in the figure can arise, wherein the shortest distance from the center to the boundaries of the zone ($r_A = 1/a\sqrt{3}$) is equal at T_{max} (or near T_{max}) to the radius of the Fermi surface $P_F = (3/8\pi)^{1/3}(Z/V)^{1/3}$, where Z is the number of electrons in the zone). An estimate of Z from the condition $r_A = P_F$ (for example, 1.11 for gadolinium), is close to the value of Z (1.15) obtained from the Jones formula [7] for the number of electrons in the first Brillouin zone.

It is clear that the characteristics of the metal at relatively low temperatures are determined not only by the electrons from the Fermi surface, but also by the electrons whose energies differ by an amount



Temperature dependence of the radius of the Fermi surface P_F and the shortest distance r_A to the boundaries of the first Brillouin zone in gadolinium.