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CONCERNING THE ANOMALIES OF THE ELECTRONIC PROPERTIES OF RARE-EARTH METALS

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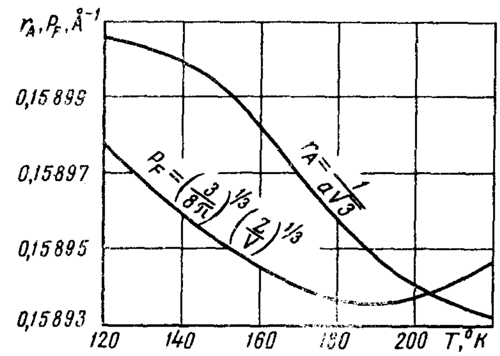
Studies of the crystal structure of rare-earth metals (gadolinium [1], terbium [2], and dysprosium [3]) in the magnetically ordered state, have revealed maxima on the plots of the atomic volume against the temperature (at $T_{max} \sim 200°K$ for Gd, 210°K for Tb, and 145°K for Dy). These maxima are connected with the different temperature dependences of the periods of the hexagonal close-packed lattices ($da/dT > 0$, $dc/dT < 0$) in the ferromagnetic region (Gd, Tb) or in the antiferromagnetic region (Dy). At approximately the same temperatures, anomalies were observed in a number of physical properties [4]: the specific magnetization, the electric resistivity, the thermal emf, etc. The purpose of the present note is to call attention to the fact that the anomalies of the physical properties of the rare-earth metals at T_{max} may be the consequence of a change in the topology of the Fermi surface [5,6], resulting from the intersection of this surface with the boundaries of the first Brillouin zone. In this case, the change in the dimensions of the Brillouin zone is due not to application of pressure (as treated in [5]), but to anomalies in the thermal deformation of the lattice.

If we confine ourselves for similicity to an isotropic dispersion law, then it is easy to verify that when the atomic volume ($V - a^2c\sqrt{3}/H$) has a nonmonotonic temperature dependence, the situation illustrated in the figure can arise, wherein the shortest distance from the center to the boundaries of the zone ($r_A = 1/a\sqrt{3}$) is equal at T_{max} (or near T_{max}) to the radius of the Fermi surface $P_F = (3/8\pi)^{1/3}(Z/V)^{1/3}$, where Z is the number of electrons in the zone). An estimate of Z from the condition $r_A = P_F$ (for example, 1.11 for gadolinium), is close to the value of Z (1.15) obtained from the Jones formula [7] for the number of electrons in the first Brillouin zone.



Temperature dependence of the radius of the Fermi surface P_F and the shortest distance r_A to the boundaries of the first Brillouin zone in gadolinium.

It is clear that the characteristics of the metal at relatively low temperatures are determined not only by the electrons from the Fermi surface, but also by the electrons whose energies differ by an amount on the order of kT. This leads to a considerable smoothing of the anomalies that are actually observed in the experiment (each of the quantities listed above has a very broad extremum near T_{max} [4]).

To check on the advanced hypothesis it would be desirable to observe metals in which such an anomaly occurs at low temperatures. This may be the case with holmium, erbium, and thulium. If the point of view presented here is correct, the anomalies should become sharper. In addition, observation of anomalies at low temperatures would make it possible to investigate the topology of the Fermi surface directly.

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COSMOLOGICAL CONSTANT AND ELEMENTARY PARTICLES

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The hypothesis that the equations of general relativity contain the cosmological constant Λ of the order of $\Lambda \cong +5 \times 10^{-56}$ cm^{-2} has been recently advanced again [1-3]. A closed world is assumed, with a contemporary radius $R_1 \sim \Lambda^{-1/2}$, a Hubble constant $H_1 \sim c\Lambda^{-1/2}$, and a density $\rho_1 \sim \Lambda c^4/G$; the presence of Λ significantly slows down the expansion during the period corresponding to the red shift z = 1.95, about which the red shifts of the absorption lines in the quasar spectrum are grouped [4]. Corresponding to the given Λ is the concept of vacuum as a medium having a density $\rho_0 = \Lambda c^4/8\pi G = 2.5 \times 10^{-29}$ g/cm^3, an energy density $\epsilon_0 = 2 \times 10^{-8}$ erg/cm^3, and a negative pressure (tension) $P_0 = -\epsilon_0 = -2 \times 10^{-8}$ $dyne/cm^3$.

How is one to visualize a theory in which such properties of the vacuum are obtained from our notions regarding elementary particles? The starting point of such a theory are the formulas that give the required order of magnitude of ϵ_0, expressed in terms of the constants m, c, ħ, and G, where m is the elementary-particle mass. Using the formulas of Eddington [5] and Dirac [6] for the quantities characterizing the contemporary universe, and the connection between these quantities and Λ, we obtain

$$\Lambda \sim G^2 m^6/\hbar^4, \quad \rho_0 \sim G m^6 c^2/\hbar^4, \quad \epsilon_0 \sim G m^6 c^4/\hbar^4. \tag{1}$$

We introduce the Compton wavelength of the elementary particle $\lambda = \hbar/mc$ and write