in the niobium-nitrogen-carbon system. The specific peculiarities of the phase with β-W lattice, whose composition in the niobium-germanium system is shifted under ordinary conditions towards niobium and corresponds approximately to $Nb_{3.3}Ge$, make such an assumption plausible. In the compound Nb_3Ge itself, quenching also increases the temperature at which the resistance begins to decrease, but, just as in the alloy investigated here, its resistivity, which begins to drop at 17°K, does not vanish even at 6°K [3]. However, we cannot exclude at present other causes of the increase in T_c, for example the influence of uncontrollable impurities entering into the alloy during the prolonged annealing. These impurities can be oxygen and nitrogen, and also silicon from the ampoule walls [4]. The authors of [2] report that the electronic specific heat of the alloy investigated by them, i.e., the coefficient γ of the linear term, is at least half as large as γ of the Nb_3Sn and V_3Si. They therefore conclude that their results do not agree with the modern microscopic theory of superconductivity, which calls for an increase in T_c with increasing electron state density N(0). However, they did not report in [2] the phase composition of the samples, so that the low values of γ may be average values taken over the entire sample, and the value of γ of the phase responsible for the high T_c may be larger than the average. In addition, when the state density is large, the simple expression relating γ with N(0) is no longer valid. It is probable that further investigations are needed to confirm the statement made in [2]. Raising the critical superconductivity temperature by two degrees is of undoubted interest, not only from the purely scientific point of view.

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THE MASS OF THE GRAVITON

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If the gravitational field is the field of the space-time curvature tensor, then the Ricci tensor R^i_k in a real field is connected with the matter tensor in a nonlocal manner [1] and (in contradiction to Einstein's equations) does not vanish outside the field sources. Consequently, there should exist in nature anomalous gravitational waves that carry the tensor R^i_k. If the anomalous gravitons have a rest mass $\mu \neq 0$, then the equations of weak anomalous waves in vacuum should be of the form $\Box f - \mu^2 f = 0$, where f is the component of the curvature tensor and $\Box$ the d'Alambert operator in flat 4-space. Let us see to what extent $\mu \neq 0$ is compatible with the metric equations of gravitation.

We assume that the equations of gravitation correspond to the principle of least action. Consequently [2]

$$D_{ik} = T_{ik} \tag{1}$$

where T_{ik} is the material tensor and D_{ik} a symmetric dynamic tensor:

$$\sqrt{-g}\, D_{ik} = 2 \frac{\delta}{\delta g^{ik}} (\sqrt{-g}\, \Lambda) ; \tag{2}$$

g is the determinant of the covariant components of the metric, and Λ is the Lagrangian of the free gravitational field containing only metric quantities. It is easy to show (by the method of [3]) that regardless of the concrete form of Λ the covariant divergence $D^i_{k;i}$ vanishes identically in accord with (2); therefore the equations $T^i_{k;i} = 0$ (which lead to the principle of a geodetic for a point) are automatically the consequence of (1). The geodetic equations follow from $T^i_{k;i} = 0$ only when the gravitational interaction is included in the material Lagrangian in accordance with the equivalence principle (in the sense of [2]), as is indeed assumed; in this case T^i_k has the usual structure.

For $\mu \neq 0$ it is necessary that the dynamic tensor of the weak field be linear relative to $\Box f - \mu^2 f$; this is accomplished by an appropriate choice of the Lagrangian. A general form of Λ, admitting of a limiting transition to the Poisson equation, has been established in [2]

$$\Lambda = \frac{1}{2\kappa_1} (R + X) , \tag{3}$$

where $\kappa_1 > 0$ is the coupling constant, R the scalar curvature, and X an invariant that decreases in a vanishing field like the n-th power ($n > 1$) of the curvature tensor (or of its covariant derivatives). For $\mu \neq 0$ it is necessary to have $n = 2$ and it is sufficient to assume that in a weak field

$$X = \frac{1}{6} (\ell_1^2 + 2\ell_2^2) R^2 - \ell_2^2 R^l_k R^k_l , \tag{4}$$

where $\ell_{1,2}$ are constants with the dimension of length; the invariant $R^{ik}_{em} R^{em}_{ik}$ (R^{ik}_{em} - Riemann tensor) is not included in (4) because in a weak field it differs from a linear combination of R^2 and $R^i_k R^k_i$ only by the divergence, which drops out identically (according to (2)) from the dynamic tensor. We choose the coordinates such that the metric of the flat 4-space coincides with γ^{ik} ($\gamma^{00} = -1$, $\gamma^{0\alpha} = 0$, $\gamma^{\alpha\beta} = \delta^{\alpha\beta}$; $\alpha, \beta = 1, 2, 3$; $x^0 = ct$). Equations (1) - (4) lead to the linearized equations of the weak field:

$$\bar{R}^l_k - \ell_2^2 \Box \bar{R}^l_k - \frac{1}{3} (\ell_1^2 - \ell_2^2) \partial^l \partial_k R = \kappa_1 (T^l_k - \frac{1}{3} \delta^l_k T^n_n), \tag{5}$$

where

$$\bar{R}^l_k = R^l_k - \frac{1}{6} R \delta^l_k , \quad \partial_k = \partial / \partial x^k , \quad \partial^l = \gamma^{lk} \partial_k ;$$

contraction of (5) yields

$$\ell_1^2 \Box R - R = \kappa_1 T_n^n . \tag{6}$$

According to (6) and (5) $\ell_{1,2}^{-1} = \mu_{1,2}$, where μ_1 and μ_2 are the masses of the gravitons corresponding to the fields R and $R_k^i - \frac{1}{4}R\delta_k^i$ respectively; consequently $\ell_{1,2}^2 > 0$, for otherwise the anomalous waves would have a superluminal group velocity.

For an empirical determination of μ_1 and μ_2, let us consider the static field of a nonrelativistic source $T_0^0 = -\rho c^2$, $T_k^i = 0$ ($i, k \neq 0$), ρ is the mass density. According to the geodetic principle [4] $g_{00} = -1 - 2c^{-2}\varphi$, where φ is the gravitational potential. From (6) and (5) (for i = k = 0) it follows that

$$\phi(x) = -\frac{\kappa_1 c^4}{8\pi} \int \frac{\rho(x')}{r_{xx'}} \left(1 + \frac{1}{3} \exp(-\mu_1 r_{xx'}) - \frac{4}{3} \exp(-\mu_2 r_{xx'})\right) d^3x', \tag{7}$$

where $r'_{xx} = |\vec{x} - \vec{x}'|$. Asymptotically, $\varphi \to (-\kappa_1 c^4/8\pi) m r^{-1}$ ($r \to \infty$; r is the distance from the mass center). There are no empirical data at present pointing to the violation of Newton's law $Gm_1m_2r^{-1}$ at large distances, and consequently, in accordance with the definition of the gravitational mass, we should put $\kappa_1 c^4 = 8\pi G$, where G is Newton's constant. As a result of such a definition of κ_1 we have $\varphi = \varphi_0 + \delta\varphi$, where φ_0 is the Newton potential, and the increment

$$\delta\phi = -\frac{G}{3} \int \frac{\rho(x')}{r_{xx'}} \left(\exp(-\mu_1 r_{xx'}) - 4\exp(-\mu_2 r_{xx'})\right) d^3x' \tag{8}$$

is the result of the nonlocal coupling between R_k^i and T_k^i, which is realized in the solutions (5) by nonlocal propagators (8) with finite radii $\ell_{1,2}$. We note now that inside a nonrelativistic source we should have $\delta\varphi \ll \varphi_0$ in order to avoid a discrepancy with Poisson's equation. Consequently both constants ℓ_1 and ℓ_2 should be small compared with the dimensions of all the nonrelativistic bodies (the factor 4 in (8) excludes the compensation of the exponentials via $\mu_1 = \mu_2$) and we unconditionally have $\ell_{1,2} \ll a$, where a is the radius of the earth (6×10^3 km). But in such a case we have in the external space $\delta\varphi \sim Gmr^{-1}\exp(-r\ell_{1,2}^{-1})$, and according to [1] we have $\delta \ll \exp(-r_M a^{-1})$, where r_M is the radius of Mercury's orbit, δ is a nonlocal correction to the rotation of its perihelion, $r_M a^{-1} \approx 10^4$, and the result strongly contradicts the empirical formula [5] $\delta \sim 10\%$, owing to the exponential decrease of $\delta\varphi$. It can be shown in general form that the degree of contradiction remains in force also when the mass spectrum is intorduced, i.e., for any quadratic structure of X. Consequently, such (logically possible) structures are empirically completely excluded from consideration (provided only $\delta > \exp(-10^4)$), and we see that $\mu \neq 0$ is incompatible with the metric theory.

If, for example, X depends only on the scalar curvature, then in a weak field [2] $3\Box\zeta - R = 0$, where $\zeta = dX/dR$. The equation has physical solutions only if $\zeta R^{-1} < \infty$ when $R = 0$ [2]. With exception of the case $(\zeta R^{-1})_0 \neq 0$ (i.e., $\mu \neq 0$), the asymptotic value of ζ satisfies the equation $\Box\zeta = 0$, corresponding to $\mu = 0$ for ζ-gravitons. The example shows that according to the metric theory real gravitational waves propagate with the speed of light and the gravitons have zero mass.

The detailed theory will be published in JETP

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DETERMINATION OF PION SCATTERING LENGTHS FROM AN ANALYSIS OF THE EXPERIMENTAL $K \to 3\pi$ DECAY DATA

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There is a rigorous theory describing reactions in which several nonresonantly-interacting particles are produced near threshold (the total kinetic energy in the final state is considerably less than the masses of the strongly-interacting particles) [1-3]. The production amplitude is represented in the form of some series in the squares of the momenta of the produced particles. The non-analytic term of this series (the terms having singularities near the physical region of the reaction), are uniquely connected with the scattering lengths of the produced particles, while the analytic ones are expanded in a Taylor series with unknown coefficients. An experimental separation of the nonanalytic terms makes it possible to determine the scattering lengths of the produced particles.

In the case of the $K \to 3\pi$ decay, the expansion of the amplitudes was verified with accuracy up to terms of order $E^{3/2}$ (E - kinetic energy released in the decay). The corresponding expressions for the probabilities of the decays $K^+ \to \pi^+\pi^+\pi^-$, $K^+ \to \pi^0\pi^0\pi^+$, and $K_2^0 \to \pi^+\pi^-\pi^0$, with allowance for the $\Delta T = 1/2$ rule, are given in [4].

The experimental data are usually presented in the form of spectra in terms of $\epsilon = 1 - (K_{12}^2/E)$ and $Z = 2(K_{13}^2 - K_{23}^2)/\sqrt{3}\,E$ divided by the phase volume; K_{il} are the relative momenta of the pions (the indices 1 and 2 pertain to identical pions or to the π^+ and π^- mesons in the $K_2^0 \to \pi^+\pi^-\pi^0$ decay), and the pion mass is equal to unity. Experimental spectra with respect to ϵ and Z of the $K^+ \to \pi^+\pi^+\pi^-$ decay and spectra with respect to ϵ of the $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$ decays are presently available. It follows from the corresponding formulas for the decay probabilities [4] that in the case of the $K^+ \to \pi^+\pi^+\pi^-$ decay the spectra with respect to ϵ and Z cannot give any information on a_0 and a_2; when $|a_0|$, $|a_2| \lesssim 1$, the terms containing a_0 and a_2 make no appreciable contribution to the distributions with respect to ϵ and Z, and these distributions are described with sufficient accuracy by the expressions

$$W^{++-}(\epsilon) = 1 + \alpha E(\epsilon - \frac{1}{2}), \quad W^{++-}(Z) = 1,$$

where α is an unknown constant connected with the analytic terms.

It is meaningless to take into account the small deviations from these formulas