light and the gravitons have zero mass.

The detailed theory will be published in JETP

[1] N. M. Polievktov-Nikoladze, ZhETF Pis. Red. 6, 531 (1967) [JETP Lett. 6, 54 (1967)].
[2] N. M. Polievktov-Nikoladze, Zh. Eksp. Teor. Fiz. 52, 1360 (1967) [Sov. Phys.-JETP 25, 904 (1967)].
[3] C. Moller, Kgl. Dansk. Vidensk. Selsk. Mat.-fys. Medd. 31, No. 14 (1959).
[4] L. D. Landau and E. M. Lifshitz, Teoria polya (Field Theory), 4th ed., Fizmatgiz, 1962 [Transl. of earlier ed: Classical Theory of Fields, Addison-Wesley, 1962].
[5] R. Dicke and H. Mark Goldenberg, Phys. Rev. Lett. 18, 313 (1967).

DETERMINATION OF PION SCATTERING LENGTHS FROM AN ANALYSIS OF THE EXPERIMENTAL $K \to 3\pi$ DECAY DATA

V. V. Anisovich and L. G. Dakhno

Submitted 26 June 1967; resubmitted 25 September 1967
ZhETF Pis'ma 6, No. 10, 907-911 (15 November 1967)

There is a rigorous theory describing reactions in which several nonresonantly-interacting particles are produced near threshold (the total kinetic energy in the final state is considerably less than the masses of the strongly-interacting particles) [1-3]. The production amplitude is represented in the form of some series in the squares of the momenta of the produced particles. The non-analytic term of this series (the terms having singularities near the physical region of the reaction), are uniquely connected with the scattering lengths of the produced particles, while the analytic ones are expanded in a Taylor series with unknown coefficients. An experimental separation of the nonanalytic terms makes it possible to determine the scattering lengths of the produced particles.

In the case of the $K \to 3\pi$ decay, the expansion of the amplitudes was verified with accuracy up to terms of order $E^{3/2}$ (E - kinetic energy released in the decay). The corresponding expressions for the probabilities of the decays $K^+ \to \pi^+\pi^+\pi^-$, $K^+ \to \pi^0\pi^0\pi^+$, and $K_2^0 \to \pi^+\pi^-\pi^0$, with allowance for the $\Delta T = 1/2$ rule, are given in [4].

The experimental data are usually presented in the form of spectra in terms of $\epsilon = 1 - (K_{12}^2/E)$ and $Z = 2(K_{13}^2 - K_{23}^2)/\sqrt{3}\,E$ divided by the phase volume; K_{il} are the relative momenta of the pions (the indices 1 and 2 pertain to identical pions or to the π^+ and π^- mesons in the $K_2^0 \to \pi^+\pi^-\pi^0$ decay), and the pion mass is equal to unity. Experimental spectra with respect to ϵ and Z of the $K^+ \to \pi^+\pi^+\pi^-$ decay and spectra with respect to ϵ of the $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$ decays are presently available. It follows from the corresponding formulas for the decay probabilities [4] that in the case of the $K^+ \to \pi^+\pi^+\pi^-$ decay the spectra with respect to ϵ and Z cannot give any information on a_0 and a_2; when $|a_0|$, $|a_2| \lesssim 1$, the terms containing a_0 and a_2 make no appreciable contribution to the distributions with respect to ϵ and Z, and these distributions are described with sufficient accuracy by the expressions

$$W^{++-}(\epsilon) = 1 + \alpha E(\epsilon - \frac{1}{2}), \quad W^{++-}(Z) = 1,$$

where α is an unknown constant connected with the analytic terms.

It is meaningless to take into account the small deviations from these formulas brought about by the terms containing a_0 and a_2, since contributions of the same order can be made by the subsequent (unaccounted-for) terms in the expansion of the amplitude (of the order of E^2 etc.).

A different situation is observed in the spectra $W^{00+}(\epsilon)$ and $W^{+-0}(\epsilon)$, the character of which differs noticeably from linear when a_0, $a_2 \neq 0$. The experimental data on these spectra are shown in Fig. 1. Comparison of the theoretical formulas [4] with the summary spectrum with respect to ϵ in the decays $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$ (Fig. 1c) makes it possible to determine the possible values of a_0 and a_2. The shaded area in Fig. 2 is the region

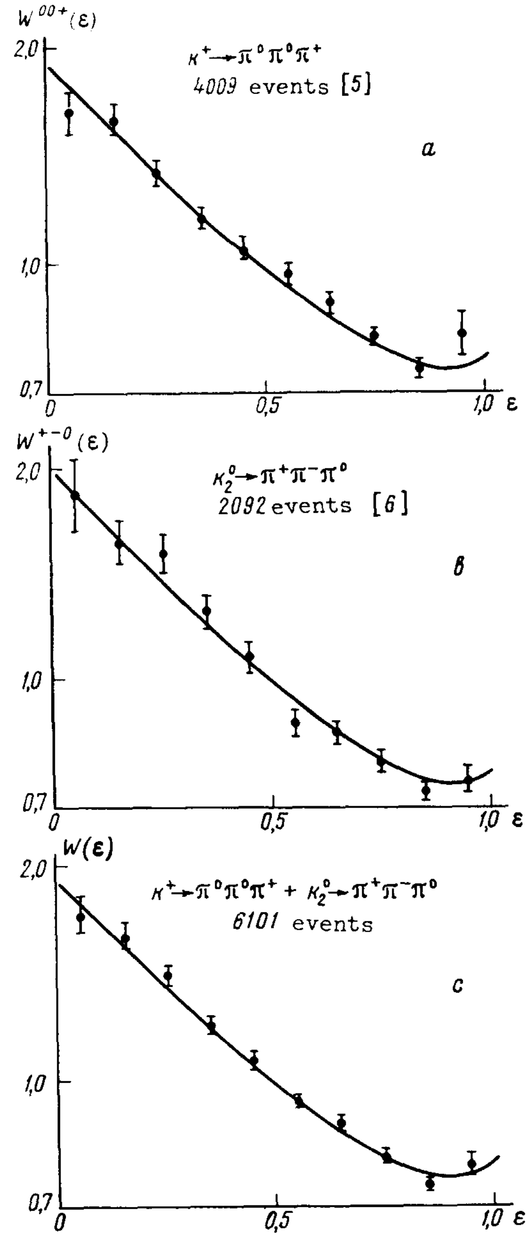

Fig. 1. Spectra with respect to ϵ in the decays $K^+ \to \pi^0\pi^0\pi^+$ [5] and $K_2^0 \to \pi^+\pi^-\pi^0$ [6]. Solid curve - theoretical for $a_0 = -0.9$, $a_2 = 0.2$ $\alpha = 1.65$.

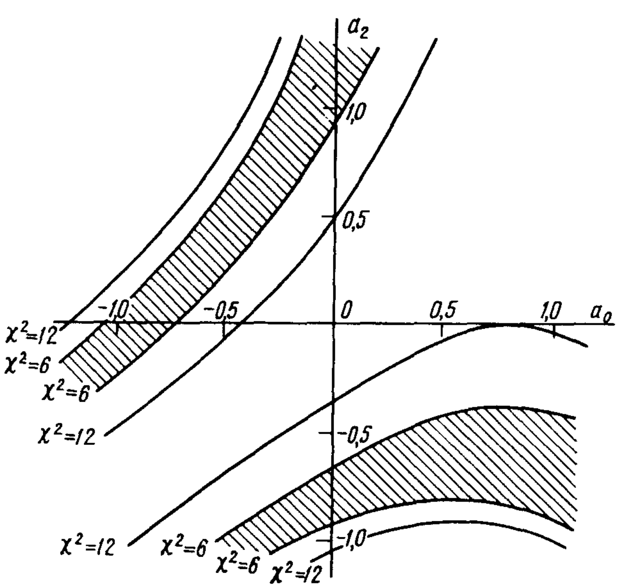

Fig. 2. Shaded areas - possible values of a_0 and a_2 in units of $h/\mu_\pi c$, obtained by comparing the theoretical formulas [4] with the spectra with respect to ϵ in the decays $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$.

of values of a_0 and a_2 for which $\chi^2 \leq 6$ (in the comparison of the corresponding theoretical formulas with the spectrum of Fig. 1c, the number of degrees of freedom is equal to 6).

The kinetic energy released in the $K \to 3\pi$ decay is not too large (~80 MeV), so that one might ask whether terms of order E^2, $E^{5/2}$, etc. could play a significant role. There are, however, considerations indicating that these terms are small. First, it follows from the theoretical formulas that the spectra $W^{++-}(\epsilon)$ and $W^{++-}(Z)$ should be described by formulas (1), which is in good agreement with the available experimental data. Second, the theoretical formulas [4] predict that when $\epsilon = 1$ there should be deviations of $W^{00+}(\epsilon)$ and $W^{+-0}(\epsilon)$ from linearity, towards larger values, for arbitrary nonvanishing a_0 and a_2. This is also observed experimentally (see Fig. 1). If the terms of order E^2, $E^{5/2}$, etc. are not small, there would be no special reason to expect such a behavior of the energy spectra $W^{++-}(\epsilon)$, $W^{++-}(Z)$, $W^{00-}(\epsilon)$, and $W^{+-0}(\epsilon)$.

The theoretical formulas [4] take into account terms of order unity, E, and $E^{3/2}$. The extent to which the region of the permissible values of a_0 and a_2 changes can be assessed by neglecting in these formulas the terms of order $E^{3/2}$ (retaining only the terms of order unity and E). It turns out that in this case the possible values of a_0 and a_2 (values leading to $\chi^2 \leq 6$) lie in the interval $0.75 \leq |a_0 - 1.1a_2| \leq 1.2$. We see thus that in this case the terms of higher order in E (of order $E^{3/2}$) have no strong influence on the result.

[1] V. N. Gribov, Zh. Eksp. Teor. Fiz. 41, 1221 (1961) [Sov. Phys.-JETP 14, 871 (1962)].

[2] V. V. Anisovich and A. A. Ansel'm, Usp. Fiz. Nauk 88, 287 (1966) [Sov. Phys.-Usp. 9, 117 (1966)].

[3] V. V. Anisovich, A. A. Anselm, and V. N. Gribov, Nuclear Physics 38, 132 (1962).

[4] V. V. Anisovich, Zh. Eksp. Teor. Fiz. 44, 1593 (1964) [Sov. Phys.-JETP 17, 1027 (1964)] V. V. Anisovich and L. G. Dakhno, Preprint, Inst. High-energy Phys., STF 67-4-K, 1967.

[5] J. K. Boeggild, K. H. Hansen, J. E. Hooper, et al., Nuovo Cimento 19, 621 (1961); G. Giacomelli, D. Monti, G. Quareni, et al., Phys. Lett. 3, 346 (1963); V. Bisi, G. Borreani, R. Cester, et al., Nuovo Cimento 35, 768 (1965).

[6] H. V. K. Hopkins, T. C. Bacon, and F. R. Eisler, Proc. Internat. Conf. on Weak Interactions, October 1965, p. 67.

CALCULATION OF THE WIDTH OF THE $f^0 \to 2\gamma$ DECAY FROM THE DISPERSION SUM RULES

G. M. Radutskii

Submitted 10 August 1967

ZhETF Pis'ma 6, No. 10, 911-913 (15 November 1967)

Great interest attaches to investigations of radiative decays of hadrons in connection with checks on the relations that follow from the assumed existence of higher symmetries. These symmetries can be very strongly violated, as indicated, for example, by the situation with the relation between $\Gamma(\eta \to 2\gamma)$ and $\Gamma(\pi^0 \to 2\gamma)$ [1]. It is therefore of interest to obtain information on the radiative decays from a reasonable dynamic model, without using the arguments of symmetry theory.

To obtain information on hadron properties, use has been made recently of dispersion sum rules derived on the basis of the dispersion relations under the assumption that the amplitudes decrease sufficiently rapidly at high energies [2]. In this paper we use the dispersion sum rule for the amplitude of the reaction $\gamma + \gamma \to \pi^0 + \pi^0$ to calculate the width