$$\frac{g_{f\pi\pi}\, g_{f\gamma\gamma}}{m_f^2} + \frac{g^2_{\rho\pi\gamma}}{4m_\rho^2} + \frac{g^2_{\omega\pi\gamma}}{4m_\omega^2} = 0,$$

which does not depend on cos φ in the approximation under consideration.

If we calculate from this equation $\Gamma(f \to 2\gamma)$, assuming that $\Gamma_{\omega\to\pi\gamma}$ = 1.15 MeV, $\Gamma_{\rho\to\pi\gamma}$ = 0.6 MeV, and $\Gamma_{f\pi\pi}$ = 100 MeV [7], then we get

$$\Gamma(f \to 2\gamma) = 0.02\ \text{MeV},$$

corresponding to approximately 0.02% of the total f-meson width.

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PHOTOPRODUCTION OF PSEUDOSCALAR MESONS ON NUCLEONS AT HIGH ENERGIES, AND REGGE POLES

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We discuss in this note the consequences of the Regge-pole hypothesis for reactions of photoproduction of pseudoscalar mesons on nucleons. According to present-day notions, the behavior of the amplitudes at high energies and low momentum transfers can be described by taking into account exchange of the nonet of vector and nonet of tensor Regge pole (exchange the Pomeranchuk vacuum pole is forbidden in the photoproduction of pseudoscalar mesons). It is assumed here that the resultant interaction constants obey the relations of exact SU(3) symmetry, and the corresponding constants of interaction of the tensor and vector mesons coincide [1] (the principle of degeneracy of vector and tensor Regge poles). An analysis of the total cross section of hadron interaction has shown that the degeneracy principle holds with good accuracy.

We then have for the π- and η-meson photoproduction amplitudes:

$$F_\lambda(\gamma p \to p\pi^0) = \kappa g_\lambda (4 + 6\beta) R_\rho \zeta_-, \quad F_\lambda(\gamma n \to n\pi^0) = \kappa g_\lambda (2+6\beta) R_\rho \zeta_-,$$

$$F_\lambda(\gamma p \to p\eta) = \kappa g_\lambda \frac{8 - 8f + 6\beta}{\sqrt{3}} R_\rho \zeta_-, \quad F_\lambda(\gamma n \to n\eta) = \kappa g_\lambda \frac{2 - 8f + 6\beta}{\sqrt{3}} R_\rho \zeta_-, \tag{1}$$

$$F_\lambda(\gamma p \to n\pi^+) = \kappa g_\lambda \sqrt{2} R_\rho (\zeta_- - 3\zeta_+), \quad F_\lambda(\gamma n \to p\pi^-) = \sqrt{2}\kappa g_\lambda R_\rho (\zeta_- + 3\zeta_+),$$

and the K-meson photoproduction amplitudes are given by

$$F_\lambda(\gamma p \to \Lambda K^+) = -\kappa g_\lambda \frac{1+2f}{\sqrt{3}} R_K(\zeta_- - 3\zeta_+),$$

$$F_\lambda(\gamma p \to \Sigma^0 K^+) = \kappa g_\lambda (1-2f) R_K(\zeta_- - 3\zeta_+), \quad (2)$$

$$F_\lambda(\gamma p \to \Sigma^+ K^0) = -2\sqrt{2}\kappa g_\lambda (1-2f) R_K \zeta_- ,$$

where λ is a definite set of helicities of the particles that take part in the reaction

$$R = \frac{1}{4\sqrt{4\pi s}} \frac{\Gamma(a+3/2)}{\Gamma(a+1)} \left(\frac{s-m^2-\mu^2}{s_0}\right)^a, \quad \zeta_\pm = \frac{1 \pm \exp(-i\pi a)}{\sin \pi a},$$

Here α is the Regge-pole trajectory ($\alpha_\rho = \alpha_\omega = \alpha_\varphi = \alpha_{K^*}$), m and μ are the masses of the final baryon and meson, s is the square of the total energy, and $s_0 = 1\ (\mathrm{GeV}/c)^2$. κ is the interaction constant at the photon vertex, and the meson-baryon interaction constants are described in the standard manner:

$$L(B\bar{B}M) = \sqrt{2}g(f\langle B[\bar{B}M]\rangle + d\langle B\{\bar{B}M\}\rangle + \beta\langle M\rangle\langle B\bar{B}\rangle),$$

$$f + d = 1.$$

An analysis of the total cross section for hadron interactions yields $\beta = 0$. We can then obtain from (1)

$$\frac{d\sigma}{dt}(\gamma p \to n\pi^+) = \frac{d\sigma}{dt}(\gamma n \to p\pi^-), \quad (3a)$$

$$\frac{d\sigma}{dt}(\gamma p \to p\pi^0) = \frac{8\sin^2(\pi a_\rho/2)}{9 - 8\sin^2(\pi a_\rho/2)} \frac{d\sigma}{dt}(\gamma p \to n\pi^+). \quad (3b)$$

Since the ρ trajectory $\alpha_\rho(t)$ passes through zero at $t = -0.6\ (\mathrm{GeV}/c)^2$, the solution (3b) predicts that the dependence of the differential cross section of the reaction $\gamma + p \to p + \pi^0$ on t will have a minimum at this point. The experimental data [2] confirm this prediction. When t = 0 we have $\alpha_\rho = 0.5$, and then

$$\frac{d\sigma}{dt}(\gamma p \to p\pi^0) = \frac{4}{5}\frac{d\sigma}{dt}(\gamma p \to n\pi^+)$$

in agreement with the experimental data. A minimum at the point $t = -0.6\ (\mathrm{GeV}/c)^2$ is predicted also for the differential cross sections for the photoproduction of η and K^0 mesons.

If the mesons are produced at angles 0 and 180°, then the baryon spin flips. The relation between the magnetic moments of the baryons and the Chew-Low statistical model [3] applied to the constants of the coupling between the Regge poles and the baryons show that in this case f/d = 2/3. It then follows from (2) that

$$\sigma(\gamma p \to \Lambda K^+)/\sigma(\gamma p \to \Sigma^0 K^+)|_{\theta=0^\circ} = 27 \quad (4)$$

in contradiction to the data of [4] which, to be sure, were obtained in the c.m.s. angle interval 25 - 40° (according to these data, the cross sections of the processes $\gamma + p \to \Lambda + K^+$ and $\gamma + p \to \Sigma^0 + K^+$ are equal).

The existence of a deep minimum at $\theta = 0°$ in the differential cross sections of the processes $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ offers evidence that at nonzero meson production angles the principal role is played by amplitudes without baryon spin flip. In this case we get for the coupling constants f/d = -2 [5] (this follows from an analysis of the total hadron-interaction cross sections). We then get in lieu of (4)

$$\frac{d\sigma}{dt}(\gamma p \to \Lambda K^+) = \frac{25}{27}\frac{d\sigma}{dt}(\gamma p \to \Sigma^0 K^+) \tag{5a}$$

in splendid agreement with the experimental data. Comparing (1) and (2) we can also obtain

$$\frac{d\sigma}{d\Omega}(\gamma p \to n\pi^+) = \frac{6}{25}\Delta(\rho, K)\frac{d\sigma}{d\Omega}(\gamma p \to \Lambda K^+), \tag{5b}$$

where the factor $\Delta(\rho, K)$, which takes into account the particle mass difference as well as the condition $\alpha_\rho \neq \alpha_{K*}$, is equal to 6 in the γ-quantum energy range 3 - 4 GeV. Therefore (5b) predicts that the cross section for the production of π^+ mesons is larger by a factor 1.5 than the cross section for the photoproduction of K^+ mesons, in agreement with the available data [4].

For the isobar-photoproduction differential cross sections, the Regge-pole model predicts

$$\frac{d\sigma}{dt}(\gamma p \to \Delta^{++}\pi^-) = 3\frac{d\sigma}{dt}(\gamma p \to \Delta^0 \pi^+), \tag{6a}$$

$$\frac{d\sigma}{dt}(\gamma p \to \Delta^+\pi^0) = \frac{2}{3}\frac{\sin^2(\pi\alpha_\rho/2)}{9 - 8\sin^2(\pi\alpha_\rho/2)}\frac{d\sigma}{dt}(\gamma p \to \Delta^{++}\pi^-), \tag{6b}$$

i.e., the differential cross section of the reaction $\gamma + p \to \Delta^+ + \pi^0$ should have a minimum at $t = -0.6\ (\mathrm{GeV/c})^2$.

One can also predict the following relation, which is valid at zero pion production angle:

$$\frac{d\sigma}{dt}(\gamma p \to p\pi^0) = \frac{16}{45}\frac{d\sigma}{dt}(\gamma p \to \Delta^{++}\pi^-),$$

and which does not contradict the known experimental data: for $E_\gamma = 3$ GeV we have $(d\sigma/dt)(\gamma p \to p\pi^0) = 1.2 \pm 0.3\ \mu b/(\mathrm{GeV/c})^2$; for γ-quantum energies in the interval 3.5 - 5.5 GeV [6] we have $(d\sigma/dt)(\gamma p \to \Delta^{++}\pi^-) = 8 \pm 5\ \mu b/(\mathrm{GeV/c})^2$. Here, however, it must be borne in mind that for $\gamma + p \to \Delta^{++} + \pi^-$ the extrapolation to the point $\theta = 0°$ is effected in a rather non-unique manner.

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TEST OF BJORKEN'S ASYMPTOTIC FORMULA IN PERTURBATION THEORY

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In a recent paper [1], Bjorken proposed a formula for the asymptotic behavior of the matrix element $M_{\mu\nu}(p', k; p, q)$:

$$M_{\mu\nu}(p', k; p, q) = -i \int dx\, e^{ikx} \langle b | T\{V_\mu^+(x),\ V_\nu^-(o)\} | a \rangle , \tag{1}$$

corresponding to the amplitude for the scattering of a W boson by an arbitrary target, due to the weak vector current V^+. According to Bjorken, when $k_0 \to \infty$ and k, p, and p' are fixed, the matrix element (1) can be expressed in terms of the simultaneous current commutator with the aid of the equality

$$\lim_{k_0 \to \infty} k_0 M_{\mu\nu}(p', k; p, q) = \int d\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}} \langle b | [V_\mu^+(o, \mathbf{x}), V_\nu^-(o)] | a \rangle . \tag{2}$$

Relation (2) can be derived in the following manner: We sum in (1) over the intermediate states and integrate with respect to k_0. We then get

$$\int d\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}} \sum_n \left\{ \frac{\langle b | V_\mu^+(o, \mathbf{x}) | n \rangle \langle n | V_\nu^-(o) | a \rangle}{k_0 + p_0' - p_{n0}} - \frac{\langle b | V_\nu^-(o) | n \rangle \langle n | V_\mu^+(o, \mathbf{x}) | a \rangle}{k_0 - p_0 + p_{n0}} \right\}. \tag{3}$$

Letting k_0 go to infinity in (3) and neglecting p_{n0} compared with k_0, we arrive at (2).

We note that in the derivation it is possible to substitute for V_μ any other operator, so that relation (2) seemingly applies to any two operators. In fact, however, it is clear that (2) is valid only when arbitrarily large intermediate energies p_{n0} play no role in the sum of (3) and in the sum produced when k_0 is taken out. This means that the amplitude $M_{\mu\nu}$ and the matrix element of the simultaneous commutator should be finite.

Let us consider by way of an example a field-theory model in which there are only nucleons and neutral pseudoscalar mesons, and the Lagrangian of their interaction is equal to

$$L = i g \bar{\psi} \gamma_5 \psi \phi . \tag{4}$$

The vector and axial currents are defined by