sec^{-1}, which is comparable with the radiative-transition probabilities ($\sim 10^8\ sec^{-1}$).

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INTERPRETATION OF ELECTRODYNAMICS AS A CONSEQUENCE OF QUANTUM THEORY

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In classical theory, action is the sum of three parts pertaining to the particles (S_p), to their interaction with the field (S_i), and to the field itself (S_f)

$$S = S_p + S_i + S_f = -mc\int dS - \frac{e}{c}\int A_i dx^i - \frac{1}{8\pi c}\int (E^2 - H^2)\, d^4x. \tag{1}$$

In quantum theory of fermions we have

$$S_p = -\int \bar{\psi}\hat{p}\psi\, d^4x, \quad S_i = -\frac{e}{c}\int \bar{\psi}\hat{A}\psi\, d^4x. \tag{2}$$

Expressing E and H in terms of A, we write $S_f = L(A_{\mu,\nu})$, where L is a quadratic expression.

In quantum theory, the interaction between the field and the vacuum leads to renormalization of the charge; this means that a new term appears in the action, of the form [1]

$$S_v = ne^2 L(A_{\mu,\nu}), \quad n = \frac{2\nu}{3\pi\hbar c}\ln \Lambda/mc. \tag{3}$$

Combining S_v with S_p and denoting the renormalized quantities by primes, we have

$$\begin{gathered} S_f' = S_f + S_v = (1 + ne^2)L(A_{\mu,\nu}) = L(A_{\mu,\nu}\sqrt{1+ne^2}) = L(A'_{\mu,\nu}), \\ A'_{\mu,\nu} = A_{\mu,\nu}\sqrt{1+ne^2},\ eA_\mu = e'A'_\mu,\ e' = e\,A_\mu/A'_\mu = e/\sqrt{1+ne^2}. \end{gathered} \tag{4}$$

Landau and Pomeranchuk [1] note that when $ne^2 \gg 1$, as $e'^2 \to n^{-1}$, "it becomes possible to neglect the effect of the free electromagnetic field in the Lagrangian." See also Fradkin [4].

Following this remark, let us consider a theory in which we start out from the expression

$$S = S_p + S_i \tag{5}$$

and the ideas related to this theory.

Physically, Eq. (5) is equivalent to assuming that there exists a "field" A (x, y, z, t) acting on the omotion of the charged particles, as can be verified by varying the particle trajectory or wave function in (5). In the non-quantum theory that follows from (5), there is no action of the field, there is no field energy, it is impossible to obtain Maxwell's equations or wave propagation, etc.

In quantum theory, on the other hand, it follows from (5) that the field A acts not only on real particles, but also on the vacuum. The theory is such that when A = 0 the energy and action of the vacuum are also identically equal to zero. In the presence of a "field" acting on the particles, and in the absence of free particles, virtual electron-positron pairs are created in the vacuum, a nonzero vacuum energy appears (which we call field energy), and a contribution to the action appears.

It is not necessary to repeat the calculations: they were already performed earlier and yielded formula (3) for S_v. We note that to obtain (3), which is the lowest term of the expansion in e^2, the present theory did not call for the quantum propagation function; we considered only the fermion loop, and are therefore justified in applying (3) to the new theory without a bare S_f. After obtaining S_v and renormalizing we arrive at the usual theory, except that the unity has been left out from $\sqrt{1 + ne^2}$ and the charge is exactly equal to $e'^2 = n^{-1}$.

An interpretation of the action of the field as a vacuum correction does not predict any new observable phenomena. It is, however, more economical than the classical theory. In the usual theory, in constructing the Lorentz-invariant expression for S, the term of the form $\int A_\mu A^\mu d^4x$ in S_f is discarded in order to reconcile the theory with the experimentally established fact that photons have no rest mass, $\mu = 0$.

The proposed interpretation of S_f' includes the quantity S_v obtained from S_i. The gauge invariance of S_i leads inevitably to gauge invariance of S_i', so that $\mu = 0$ is the consequence of the theory, and not an additional requirement taken from experiment.

The present note is a generalization of an idea developed for gravitation theory by A. D. Sakharov [2] to the case of the electromagnetic field. Sakharov regards the vacuum correction to be the expression for the action S_{fg}, that depends on the curvature R of the space, usually written in the form

$$S_{fg} = \frac{c^3}{16\pi G} \int R dv. \tag{6}$$

He obtains in quantum theory, in order of magnitude,

$$S_{vg} = \hbar^{-1}\Lambda^2 \int R \, dv. \tag{7}$$

Here, just as in (3), Λ is the limiting momentum (cutoff momentum).* Comparing (6) and (7), we obtain an expression for G, or more accurately for the observable G', analogous to $e'^2 = n^{-1}$, namely

$$G' = \hbar c^3/\Lambda^2. \tag{8}$$

Eliminating Λ from the expressions for G' and e', we get

$$e'^2 = \hbar c\, 3\pi [\, \nu \ln (\hbar c / G m^2\,)\,]^{-1}\,. \tag{9}$$

This relation, which is given in [1], agrees with experiment when the number of types of charged particles is $\nu \simeq 13$. It was assumed in [1] that the limit of the electrodynamics and the momentum Λ are determined by the gravitational interaction of the virtual particles.

The gist of the proposed interpretation is that in electrodynamics as well as in gravitation the field terms represent vacuum correction with a total cutoff momentum Λ, which is an externally specified constant of the theory.

The field itself is primarily a factor that acts only on the particle motion (in the case of gravitation - as a result of space curvature), but has no energy of its own.

Assuming $\Lambda \gg m$, it follows automatically from the proposed interpretation that the vector (electromagnetic) interaction has a logarithmically small constant, i.e., a small but not very small charge, and the gravitational interaction is small like m^2/Λ^2, i.e., it is quite small.

The notion that quanta are electron-positron pairs (for references see [3]) now acquires a new and not oversimplified interpretation.

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* It is possible that the diagram technique and the theory of the tensor field h_{ik} in flat space can also be used in gravitation.