INSTABILITY OF ELASTIC OSCILLATIONS IN PIEZOELECTRICS IN ALTERNATING ELECTRIC FIELDS

A. A. Chaban
Acoustics Institute, Moscow
Submitted 19 September 1967; resubmitted 7 October 1967
ZhETF Pis'ma 6, No. 11, 967-970 (1 December 1967)

The instability of elastic lattice oscillations, induced in piezoelectric semiconductors by a constant electric field, has by now been thoroughly investigated [1]. It will be shown below that strong nonlinearity of the electric properties of semiconductors [2] can lead to amplification and generation of standing sound waves of specified frequency under the influence of an external alternating electric field that exceeds a certain threshold value. An instability of this type can really be observed not only in semiconductors, but also in piezodielectrics, as a result of corresponding nonlinear effects. The frequency of the generated and amplified standing sound waves should be half the frequency of the alternating electric field. This effect is analogous to the instability of elastic oscillations in ferrites in the presence of an alternating magnetic field [3].

Let us consider first a semiconducting piezoelectric crystal. The nonlinearity of the electric properties is described in this case by the equation [1]

$$-\frac{\partial^2 D}{\partial x\, \partial t} = \mu \frac{\partial}{\partial x}\{(q n_0 - f\frac{\partial D}{\partial x})E\} - D_n f \frac{\partial^3 D}{\partial x^3}. \tag{1}$$

Here E is the electric field intensity, $D = \epsilon E + e(\partial u/\partial x)$ the electric induction, u the mechanical displacement, ϵ the dielectric constant, e the piezoelectric constant, -q the electron charge, D_n the diffusion coefficient, μ the mobility, n_0 the number of electrons in the conduction band, and f a factor characterizing the capture of the electrons by the traps. The elasticity-theory equation takes the form [1]

$$\rho \frac{\partial^2 u}{\partial t^2} = C \frac{\partial^2 u}{\partial x^2} - e \frac{\partial E}{\partial x}, \tag{2}$$

where ρ is the crystal density and C the modulus of elasticity.

Assume that a standing elastic wave of frequency ω is excited in the crystal. Furthermore, we assume that an external alternating field $E_0 \cos(2\omega t + \delta)$ is applied to the crystal. If we retain in the nonlinear part of (1) only the terms that vary with time at the frequency ω (the other Fourier components, connected with the generation of harmonics, are disregarded), then for an elastic wave

$$u = u_0 \cos(\omega t + a) \cos(kx + \beta) \exp(-\kappa t)$$

we can write for the time-dependent standing-wave damping coefficient, when $\kappa \ll \omega$,

$$\kappa = \frac{e^2}{\epsilon C}\omega_c\left[1 - \frac{\mu f E_0}{2s}\cos(\delta - 2\alpha)\right]\left\{1 + \left(\frac{\omega_c}{\omega} + \frac{\omega}{\omega_D}\right)^2 - \left(\frac{\mu f E_0}{2s}\right)^2\right\}^{-1}, \qquad (3)$$

where $\omega_c = \delta/\epsilon$, $\omega_D = s^2/fD_n$, and $s = \sqrt{C/\rho}$. If E_0 is sufficiently large and $(\delta - 2\alpha)$ is of appropriate magnitude, we can obtain $\kappa < 0$, i.e., the sound is amplified. Although the amplification by an alternating electric field is of approximately the same magnitude as that by a constant field, the essential difference between the two must be emphasized. An alternating electric field acts only on one lattice-vibration mode, and can be used to generate a signal of specified frequency, and not merely a noise signal. Furthermore, if $\omega \gg \omega_c$, the problem of electric contacts is eliminated, since the external field can be produced by a resonator and the crystal placed inside the resonator.

Let us consider now a nonconducting piezoelectric crystal. Confining ourselves to the one-dimensional case, as in the preceding calculation, let us study the behavior of the standing wave under the influence of an alternating electric field. We include in the expression for the considered component of the stress tensor T and the electric induction D terms of second order in $\partial u/\partial x$ and E (cf. [4])

$$\begin{cases} T = C\dfrac{\partial u}{\partial x} - eE + 2\eta\dfrac{\partial u}{\partial x}E + QE^2 + \theta\left(\dfrac{\partial u}{\partial x}\right)^2; \\ D = \epsilon E + e\dfrac{\partial u}{\partial x} - 2Q\dfrac{\partial u}{\partial x}E + OE^2 - \eta\left(\dfrac{\partial u}{\partial x}\right)^2. \end{cases} \qquad (4)$$

Here η is the quadratic piezoelectric coefficient, O the electrooptical constant, Q the electrostriction constant, and θ the anharmonicity constant. Simultaneous solution of the Poisson equation, the elasticity-theory equation, and (4) leads to the following expression for the time-dependent damping coefficient

$$\kappa = \frac{\omega E_0 \sin(\delta - 2\alpha)}{\rho s^2}\left(\eta - \frac{2eQ}{\epsilon} + \frac{e^2 O}{\epsilon^2}\right), \qquad (5)$$

where the quantities δ and α have the same meaning as in (3). Thus, with an appropriate value of $(\delta - 2\alpha)$ and a sufficiently large E_0 we should observe amplification of the sound (as soon as the quantity $-\kappa$ exceeds the usual damping due to the lattice anharmonicity).

The main contribution to (5) is apparently made by the first term. Unfortunately, we know of no measurements of η and we are forced to confine ourselves to an estimate of the next term, which is connected with electrostriction. For $\omega \simeq 10^{11}$ sec^{-1}, $e \simeq 10^5$ cgs esu, $Q \simeq 10$ cgs esu, $\epsilon \simeq 10$, $E_0 \simeq 10^4$ V/cm we get $|\kappa|/5 \sim 10$ cm^{-1}. Thus, at least at low temperatures, we can experimentally observe amplification and generation of standing sound waves in nonconducting piezoelectrics. (In ferroelectrics, the electrooptical effect may give a contribution of the same order as electrostriction.)

It is easy to ascertain that the sound instability in question is connected with the following circumstance. The nonlinear interaction between a traveling elastic wave and an alternating electric field of double the frequency gives rise to a force that excites resonantly an elastic wave of the same frequency as the original one, but propagating in the opposite direction (for semiconductors this question was discussed in [5]). Then the presence of two elastic waves traveling in opposite directions and forming a standing wave can lead to amplification of both waves under the influence of the electric field. The corresponding conditions were obtained above.

[1] D. L. White, J. Appl. Phys. 33, 2547 (1962).

[2] B. Tell, Phys. Rev. 136, A772 (1964); C. Elbaum and R. Truell, Appl. Phys. Lett. 4, 212 (1964); H. Kroger, Appl. Phys. Lett. 4, 190 (1964).

[3] R. L. Comstock and R. C. Le Craw, Phys. Rev. Lett. 10, 219 (1963).

[4] S. I. Pekar, Zh. Eksp. Teor. Fiz. 49, 621 (1965) [Sov. Phys.-JETP 22, 431 (1965)].

[5] A. A. Chaban, Fiz. Tverd. Tela 9, 3334 (1967) [Sov. Phys.-Solid State 9, (1968), in press].

Article by A. A. Chaban, Vol. 6, No. 12.

On page 382, top line, "$\omega_c = \delta/\epsilon$..." should read "$\omega_c = \sigma/\epsilon$, σ - conductivity..."

On page 382, line 5 from bottom, "$|\kappa|/5 \sim 10\ cm^{-1}$..." should read "$|\kappa|/s \sim 10\ cm^{-1}$..."