MASS AND CHARGE RENORMALIZATION CONSTANTS IN QUANTUM ELECTRODYNAMICS WITHOUT A BARE MASS

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1. The purpose of the article is to show that the situation with the renormalization constants in quantum electrodynamics is substantially altered if the bare mass m_0 is not introduced into the theory and the electron mass m is defines as the purely dynamic effect. Then m turns out to be linearly dependent on the cutoff parameter Λ with a non-analytic dependence on the coupling constant α, while the charge renormalization is finite and the well-known difficulty with the "nullification of the charge" is eliminated.

This result is connected with a definite departure beyond the framework of ordinary perturbation theory, although perturbation theory is used in part for its proof.

2. When $m_0 = 0$, the Lagrangian of quantum electrodynamics, which is the sum of the ordinary unperturbed Lagrangian L_0 and the interaction Lagrangian L_i, does not contain a mass term and has γ_5-invariance. The mass, however, can results as a dynamic effect in a system with "violated γ_5-invariance" [1].

The mass can be defined with the aid of the well-known counterterm method. We shall construct a perturbation theory, describing the unperturbed system not by the Lagrangian L_0, but by $L_0' = L_0 - m\bar{\psi}\psi$, and regard $L_i' = L_i + m\bar{\psi}\psi$ as the interaction Lagrangian. For further operations, it is convenient first to redefine $m \to \mu$ in the counterterm, writing $L_i' = L_i + \mu\bar{\psi}\psi$, and then again put $\mu = m$ after completing all the transformations.

In order that the parameter m introduced into L_0' have the meaning of a renormalized mass, the parameter μ in L_i' should be defined such that the mass renormalization resulting from the action of the modified interaction Lagrangian L_i' vanish. In other words, μ must be chosen such that the pole of the electronic Green's function does not shift when L_i' is taken into account, and coincides with the pole $p^2 = m^2$ of the free Green's function. Since, on the other hand, $\mu = m$, we obtain an equation for determining m.

In this scheme, the electronic Green's function is determined by means of the usual series of self-energy Feynman diagrams plus a series of diagrams obtained from the data by making all sorts of insertion into the electronic lines of the element containing the term $\mu\bar{\psi}\psi$. With this, μ should be written in the form of the sum $\mu = \mu_1 + \mu_2 + \ldots$ and the term μ_1 must be so disposed as to compensate for the mass renormalization occurring in second order, with μ_2 producing the same effect in fourth order, etc.

It is easy to verify by means of a direct estimate of the character of the divergence of the self-energy diagrams that the μ_n have the following structure (we assume that the charge renormalization has been effected and denote by α the renormalized coupling constant)

$$\mu_1 = m\alpha\left(a_1 \ln\frac{\Lambda}{m} + b_1\right), \quad a_1 = \frac{3}{2\pi},$$

$$\mu_2 = m\alpha^2\left(a_2 \ln^2\frac{\Lambda}{m} + b_2 \ln\frac{\Lambda}{m} + c_2\right),$$

$$\mu_n = m\alpha^n \left(a_n \ln^n \frac{\Lambda}{m} + b_n \ln^{n-1} \frac{\Lambda}{m} + \dots\right),$$

where a_n, b_n, ... are numbers. Assuming that $\ln(\Lambda/m)$ is large, we retain at first only the higher powers of the logarithm in each of the μ_n. Equating m to the sum of all the μ_n, we obtain

$$m \sum_{n=1}^{\infty} a_n \left(\alpha \ln \frac{\Lambda}{m}\right)^n = m. \tag{2}$$

Besides the trivial solution m = 0, this equation admits of a non-zero solution

$$m = \Lambda \exp\left(-B/\alpha\right), \tag{3}$$

where B is a number defined by the relation

$$\sum_{n=1}^{\infty} a_n B^n = 1. \tag{4}$$

To determine B it is necessary to find all the a_n. If we include in the sum of (4) only the first term with $a_1 = 3/2\pi$, then we obtain $B = 2\pi/3$. Preliminary estimates show that all the a_n are positive and decrease with increasing n; consequently, their inclusion will decrease the value of B. This allows us to write the inequalities

$$0 < B < \frac{2\pi}{3}. \tag{5}$$

The question of the uniqueness of the solution will be left open for the time being.

3. It is easy to show that the charge renormalization constant Z_3, as a function of the cutoff parameter Λ and the renormalized mass m, retains in our theory the same form as in the ordinary theory. In the same "logarithmic" approximation in which Eq. (2) and the solution (3) were obtained, retaining in each order with respect to the coupling constant only the higher powers of $\ln(\Lambda/m)$ (which coincide with the power of the coupling constant), the charge renormalization took the form [2]

$$\alpha = \alpha_0 \left(1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{-1}, \tag{6}$$

$$Z_3 = \frac{\alpha}{\alpha_0} = \left(1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{-1} = 1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2} \tag{7}$$

(α_0 is the non-renormalized interaction constant).

Substituting expression (3) for m in (7), we get

$$Z_3 = \frac{\alpha}{\alpha_0} = 1 - \frac{2}{3\pi} B. \tag{8}$$

The charge renormalization is thus finite. From the estimate (5) for B it follows that Z_3

lies in the range

$$\frac{5}{9} < Z_3 < 1. \tag{9}$$

The well-known difficulty with the "nullification of the charge" [3] is thus eliminated.

4. Expression (3) for the field mass does not admit of expansion in powers of α. Although the counterterm method employed by us is based on a diagram technique, it represents a definite departure outside the framework of consistent perturbation theory. The reason is that in the construction of Eq. (2), which defines the mass of α, we first consider m as an independent parameter and we do not assume it to be expandable in a series in the interaction constant. In particular, the representation of the counterterm μ in the form of a sum $\mu = \mu_1 + \mu_2 + \ldots$, whose terms are defined by relation (1), is not a consistent expansion in powers of α, since the dependence of α also enters via m. Substitution of the solution (3) in (1) shows that all the μ_n are of the same order of magnitude.

5. In the "logarithmic" approximation employed above, we discard an infinite series of "nonprincipal" terms. If we retain in expressions (1) for μ_n not only the principal terms but also the succeeding powers of $\ln(\Lambda/m)$, then the numerical coefficient B in (3) will be replaced by the series $B + \alpha B_1 + \alpha^2 B_2 + \ldots$, and consequently (3) is replaced by

$$\begin{aligned} m &= \Lambda \exp\left(-\frac{B}{\alpha} - B_1 - \alpha B_2 - \ldots\right) = \\ &= \Lambda \exp(-B_1)\exp\left(-\frac{B}{\alpha}\right)\{1 - \alpha B_2 + \ldots\}. \end{aligned} \tag{10}$$

Since the perturbation-theory series in quantum electrodynamics are presumably asymptotic series, it is natural to expect the series in the curly brackets in (10) to be ssymptotic and, taking the smallness of the parameter α into account, it is accurate enough to retain the first term only. In this case replacement of the result (3) by expression (10) does not change qualitatively the foregoing conclusions.

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COMPENSATION OF A BALLOON INSTABILITY MODE OF A PLASMA IN A TOROIDAL SYSTEM

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The magnetic force lines in toroidal systems for plasma containment are convex on the outside and concave on the inside. Therefore, at a finite plasma pressure, the translational