lies in the range

$$\frac{5}{9} < Z_3 < 1. \tag{9}$$

The well-known difficulty with the "nullification of the charge" [3] is thus eliminated.

4. Expression (3) for the field mass does not admit of expansion in powers of α. Although the counterterm method employed by us is based on a diagram technique, it represents a definite departure outside the framework of consistent perturbation theory. The reason is that in the construction of Eq. (2), which defines the mass of α, we first consider m as an independent parameter and we do not assume it to be expandable in a series in the interaction constant. In particular, the representation of the counterterm μ in the form of a sum $\mu = \mu_1 + \mu_2 + \ldots$, whose terms are defined by relation (1), is not a consistent expansion in powers of α, since the dependence of α also enters via m. Substitution of the solution (3) in (1) shows that all the μ_n are of the same order of magnitude.

5. In the "logarithmic" approximation employed above, we discard an infinite series of "nonprincipal" terms. If we retain in expressions (1) for μ_n not only the principal terms but also the succeeding powers of $\ln(\Lambda/m)$, then the numerical coefficient B in (3) will be replaced by the series $B + \alpha B_1 + \alpha^2 B_2 + \ldots$, and consequently (3) is replaced by

$$\begin{aligned} m &= \Lambda \exp\left(-\frac{B}{\alpha} - B_1 - \alpha B_2 - \ldots\right) = \\ &= \Lambda \exp(-B_1)\exp\left(-\frac{B}{\alpha}\right)\{1 - \alpha B_2 + \ldots\}. \end{aligned} \tag{10}$$

Since the perturbation-theory series in quantum electrodynamics are presumably asymptotic series, it is natural to expect the series in the curly brackets in (10) to be ssymptotic and, taking the smallness of the parameter α into account, it is accurate enough to retain the first term only. In this case replacement of the result (3) by expression (10) does not change qualitatively the foregoing conclusions.

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COMPENSATION OF A BALLOON INSTABILITY MODE OF A PLASMA IN A TOROIDAL SYSTEM

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The magnetic force lines in toroidal systems for plasma containment are convex on the outside and concave on the inside. Therefore, at a finite plasma pressure, the translational instability that develops without distortion of the magnetic force lines may also be accompanied by a balloon instability, which is manifest in a bulging of the force tubes on the outer side of the torus.

At a finite plasma pressure, however, there exists also in toroidal systems another, useful balloon effect. It consists in a shift of the entire plasma pinch towards the outer wall of the torus. Since the internal magnetic surfaces are shifted more strongly than the external ones, a situation arises wherein the toroidal system has an averaged "magnetic well" - the average magnetic field on the outer surfaces is larger than on the inner ones. We shall show below, using as an example a two-turn stellarator with circular magnetic axis and a system with a helical magnetic axis, that the deepening of the magnetic well due to the plasma pressure cancels out the balloon instability if ideal plasma conductivity is assumed.

An expression for the second derivative of the volume with respect to the longitudinal magnetic flux $V''(\Phi)$, which characterizes the depth of the magnetic well, is given in [1]. It is a linear function of the parameters α_1, α_2, α_3, and α_4 of the magnetic surfaces. The values of these parameters in the absence of a plasma were determined in [1]. The corrections for the plasma pressure can be determined by a perturbation method, as was done in Sec. 11 of [1], where the shift of the magnetic axis was determined for a specified external magnetic surface. In contrast to this calculation, we now must assume not a fixed external surface, but a fixed magnetic axis (since the formulas for V'' have been written out in a coordinate system in which the magnetic axis coincides with a coordinate). For this purpose it is necessary to set the constants C_1 and C_2 in the expression for the pressure-related transverse magnetic field equal to zero.

A straight forward but somewhat laborious calculation leads to the following expression for V'' in a two-turn stellarator

$$V'' = -\frac{V'}{\pi B_0\sqrt{1-\epsilon^2}}\left\{\frac{11}{2}K^2 - \epsilon^2\delta'^2 - \frac{K^2\pi p'(8-6\epsilon^2-\epsilon^4)}{B_0\delta'^2\epsilon^4\sqrt{1-\epsilon^2}}\right\} + \frac{p'V'}{B_0^2} \equiv V''_0 + V''_p . \quad (1)$$

Here B_0 is the magnetic field on the axis, K the curvature of the axis, ϵ a parameter connected with the ratio of the semiaxes ℓ_1 and ℓ_2 of the elliptic cross sections of the magnetic surfaces, $\epsilon = (\ell_1^2 - \ell_2^2)/(\ell_1^2 + \ell_2^2)$, $\delta' \equiv kN$, and N is the total number of turns executed by the elliptic cross section in going around the torus. The pressure in the vicinity of the axis is taken in the form of an expansion $p = p_0 + p'\Phi$, so that $p' < 0$. We see that $V''_p < 0$, corresponding to a deepening of the magnetic well as a result of the plasma pressure.

To compare the effect of the deepening of the well with the effect of the balloon instability, we shall use a general criterion obtained by Solov'ev [2] for the stability of a plasma against local perturbations in the vicinity of the magnetic axis

$$\frac{1}{4B_0^2}\left(\frac{\chi''}{V'}\right)^2 + \frac{1}{|\nabla\Phi|^2}\left\{p'\frac{V''}{V'} - \frac{p'^2}{B_0^2} - \left\langle\frac{(\rho\partial j_s/\partial\rho)^2}{|\nabla\Phi|^2}\right\rangle\right\} \geqslant 0. \tag{2}$$

Here j_s is the longitudinal current density and ρ the distance from the axis; the primes denote differentiation with respect to Φ and the angle brackets averaging over the volume of the layer contained between two neighboring magnetic surfaces. The first term, which contains the second derivative of the flux χ, characterizes the stabilizing role of "shear." The second and third terms correspond to translational instability, and the fourth to the balloon instability modes. We note that the last term in the expression (1) for V", which does not depend on the curvature, describes a trivial deepening of the magnetic well owing to the diamagnetism of the plasma contained by the magnetic field. This terms is compensated by the second term in the curly brackets of formula (2) and thus drops out of the stability criterion.

The expression for j_s in the vicinity of the magnetic axis is given in [1] (formula 11.46). The use of this expression yields, after suitable averaging,

$$\left\langle\frac{(\rho\partial j_s/\partial\rho)^2}{|\nabla\Phi|^2}\right\rangle = \frac{2k^2p'^2}{B_0^2\delta'^2\epsilon^4}(4\sqrt{1-\epsilon^2}+\epsilon^2). \tag{3}$$

If we now combine this term in criterion (2) with the term containing V_p'', then the stability criterion takes the form

$$\frac{1}{4B_0^2}\left(\frac{\chi''}{V'}\right)^2 + \frac{1}{|\nabla\Phi|^2}\left\{p'\frac{V_0''}{V'} + \frac{k^2p'^2}{B_0^2\delta'^2\epsilon^4}f(\epsilon)\right\} \geqslant 0, \tag{4}$$

where

$$f(\epsilon) = \frac{8-6\epsilon^2-\epsilon^4}{1-\epsilon^2} - 8\sqrt{1-\epsilon^2} - 2\epsilon^2 = 4\epsilon^2\left(1+\frac{\epsilon^2}{2}+\ldots\right). \tag{5}$$

We see that $f(\epsilon) > 0$, i.e., the effect of deepening of the magnetic well is even larger than the effect of the balloon instability.

In the case of a system with helical magnetic axis, having a curvature k and a torsion κ, the last term in the curly brackets of criterion (4) is replaced by

$$\frac{k^2p'^2}{B_0^2\kappa^2(1+\epsilon)}\left[\frac{2-\epsilon}{1-\epsilon^2} - \frac{2}{\epsilon}\left(1-\sqrt{\frac{1-\epsilon}{1+\epsilon}}\right)\right] = \frac{k^2p'^2}{B_0^2\kappa^2}\epsilon^2\left(1-\frac{5}{4}\epsilon+\ldots\right) > 0, \tag{6}$$

Consequently, a sufficient stability criterion for both systems is the presence of a magnetic well in the vacuum magnetic field. No limitation is imposed here at all on the plasma pressure. Moreover, if we have in the vacuum field a "magnetic hill" in lieu of a magnetic well ($V_0'' > 0$), then the plasma can become stable even in this case if the pressure is high enough.

Let us denote by Δ the relative height of the magnetic hill, $\Delta = V_0''\Phi/V'$, and by ι the angle of the rotational transformation ($\iota = N\epsilon^2/2$ and $\iota = \kappa L$, where L is the length of the magnetic axis of the system, for the two systems, respectively). Then the ratio $\beta = 2p'\Phi/B_0^2$ of the plasma pressure to the magnetic-field pressure at which self-stabilization of the plasma takes place in the absence of shear is given by the condition

$$\beta > \frac{2\iota^2\Delta}{K^2L^2\epsilon^2} \cong \frac{2\Delta}{\epsilon^2}\left(\frac{\iota}{2\pi}\right)^2. \qquad (7)$$

The foregoing examples show that the balloon-instability criteria obtained by solving model problems [3,4] do not give a correct idea of the role of this instability in real toroidal systems. It was already shown earlier [5-7] that in the Tokamak system there are automatically produced conditions under which the balloon instability becomes stabilized. It follows from the foregoing results that self-stabilization of a plasma as the result of the toroidal effect is realized also in systems of the stellarator type.

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NEUTRAL CURRENTS IN WEAK INTERACTION THEORY

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In an earlier paper [1] we considered weak interaction between hadrons and neutral lepton currents that arise effectively in the theory with charged currents in second order in the weak interaction constant G. It was shown on the basis of current algebra that the momenta of the virtual hadrons are not cut off by strong interactions, so that the effective interaction constant of the hadrons with the neutral lepton current is $G^0_{\text{eff}} \sim G^2\Lambda^2$, where Λ is the cutoff due to weak or electromagnetic interactions.

This result was based, besides the assumption on the commutation relations between the current components, on additional hypotheses concerning the asymptotic behavior of certain amplitudes at large momenta (for example, an amplitude proportional to the divergence of the axial current). In the present note, assuming that the asymptotic behavior of the amplitude

$$M^{\alpha\beta}_{\mu\nu}(p', k; p, q) = i\int d^4x\, e^{ikx}\langle a|T\{j^\alpha_\mu(x),\ j^\beta_\nu(o)\}|b\rangle \qquad (1)$$