of the term $\epsilon_{jk\ell}$ in (11). In [4,5] they proposed models in which the contribution of this term compensated, in the radiative corrections to β decay, the divergence due to the remaining terms. In our case the divergences remain in these models. For example, in the case of the model of [4] with basic particles Ξ^-, Ξ^0, Λ, and S (S - unitary singlet) we obtain in lieu of (13) a value that is four times larger, and in the case of the SUB model [5] the result is indeterminate. We note that by writing the current in the form V + ρA with different values of ρ it is possible to eliminate the quadratic divergence in a process due to an axial current (for example, $K_2^0 \to \mu^+ + \mu^-$), but this cannot be done for a process due to a vector current (for example, $K^+ \to \pi^+ + e^+ + e^-$).

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HYSTERESIS OF CYCLOTRON ABSORPTION BY THE CARRIERS IN A NONPARABOLIC BAND

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We consider an electron with isotropic but nonparabolic dispersion $\epsilon(p)$ in a magnetic field $\vec{H} \parallel z$. In momentum space, it executes periodic motion along the circle obtained when the sphere $\epsilon(p)$ = const crosses the plane p_z = const. The momentum components p_x and p_y oscillate in time with amplitude $p_\perp$ and frequency $\omega_H(p)$, with [1]

$$p_\perp = \sqrt{p^2 - p_z^2}, \quad \omega_H(p) = \frac{eH}{c}\frac{1}{p}\frac{\partial \epsilon}{\partial p}. \tag{1}$$

For a nonparabolic band ω_H depends on p, i.e., on the oscillation amplitude $p_\perp$. The electron can therefore be regarded as a nonlinear oscillator. It is known that the amplitude of forced oscillations is, in general, a non-unique function of the frequency of the driving force [2], and that hysteresis is observed on going through resonance [3]. It is therefore natural to expect in cyclotron resonance a similar effect for the dependence of the absorbed power on the electromagnetic-field frequency.

The equations of motion of the electron (for simplicity $p_z = 0$, $p_\perp = p$)

$$\begin{cases} \dfrac{dp_x}{dt} = \omega_H(p) p_y - \dfrac{p_x}{\tau} + eE_x(t) \\ \dfrac{dp_y}{dt} = -\omega_H(p) p_x - \dfrac{p_y}{\tau} + eE_y(t), \end{cases} \tag{2}$$

where τ is the relaxation time and $\vec{E}(t)$ is the high-frequency electric field. Regarding $\vec{E}(t)$ as harmonic (of frequency ω and of sufficiently small amplitude E), we can obtain the forced oscillations with period $2\pi/\omega$ in the small-nonlinearity approximation by the method of slowly varying amplitudes and phases [3].

The forced oscillation turns out to be harmonic and, as expected, the dependence of the amplitude p and of the phase φ of the forced oscillation on E and ω is given by the same relations as for a linear oscillator, the only difference being that in these relations ω_H must be replaced by $\omega_H(p)$:

$$p^2[\omega - \omega_H(p)]^2 + \frac{p^2}{\tau^2} = (eE)^2$$
$$\mathrm{tg}\,\phi = [\,\omega - \omega_H(p)\,]\tau . \tag{3}$$

It is easy to verify by direct substitution that this solution is exact for a circularly polarized field.

We now assume for a small nonlinearity

$$\omega_H(p) = \omega_H\left(1 + \frac{p^2}{p_0^2}\right), \quad \omega_H = \frac{eH}{mc}, \tag{4}$$

where p_0 characterizes the nonparabolicity of the band and m is the effective mass at the bottom of the band. Solution of the first equation in (3) yields $p(\omega)$. The maximum of this quantity, just as for linear oscillations, is $p_{max} = eE\tau$. It is easy to show that if the field exceeds a certain critical value $E > E_c$, where

$$(eE_c)^2 = \frac{4p_0^2}{\omega_H\tau^3}, \tag{5}$$

there exists an interval of frequencies ω near ω_H where each ω corresponds to three values of p, two of which are stable and one unstable. It is clear from (3) that the non-uniquness of $p(\omega)$ results in non-uniqueness of $Q(\omega)$ and in hysteresis of $Q(\omega)$ on going through resonance.

Let us estimate the critical amplitude of the oscillations p_c, i.e., p_{max} at $E \simeq E_c$. From (5) we get

$$p_c \simeq p_0(\omega_H\tau)^{-1/2}. \tag{6}$$

Since observation of a resonance requires that

$$\omega_H\tau \gg 1, \tag{7}$$

it follows that $p_c \ll p_0$ and the small nonlinearity conditions are satisfied under hysteresis conditions. In order for the classical treatment of the electron motion to be valid it is necessary to have

$$\frac{p_c^2}{2m} \gg \hbar\omega_H, \tag{8}$$

which imposes an upper limit on the magnetic field. The last three equations yield ultimately

$$\frac{\hbar}{\tau} << \hbar\omega_H << \left(\frac{\hbar}{\tau}\frac{p_0^2}{2m}\right)^{1/2}. \qquad (9)$$

which is attainable in a sufficiently pure material with not too large a nonlinearity.

Assuming that the dispersion law is as given by Kane [4], we get

$$\frac{p_0^2}{2m} = \frac{9}{64}\frac{\hbar^4 E_G^3}{m^2 P^4}, \qquad (10)$$

where E_G is the width of the forbidden band and P is the matrix element of the spin-orbit interaction. To obtain an idea of the order of magnitude of the quantities, we assume numerical values for n-InSb (the quantities without dimensions correspond to atomic units):

$$E_G = 0{,}88\cdot 10^{-2} = 0{,}23 \text{ eV},\ P = 0{,}44,\ m = 1{,}3\cdot 10^{-2},$$
$$\tau = 10^{-11} \text{ sec}$$

then

$$\frac{p_0^2}{2m} \simeq 2\cdot 10^{-2} \simeq 0{,}5 \text{ eV},\quad \frac{\hbar}{\tau} \simeq 2\cdot 10^{-6} = 0{,}6\cdot 10^{-4} \text{ eV}.$$

Assuming H = 500 Oe we get $\omega_H = 10^{12}$ sec^{-1} (λ = 2 mm) and $\hbar\omega_H = 0.6 \times 10^{-3}$ eV, so that (9) is satisfied. At these values we obtain $E_c \simeq 200$ V/cm.

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CAUSE OF BEATS IN THE OBSERVATION OF DE HAAS - VAN ALPHEN EFFECT IN METALS OF THE BISMUTH TYPE

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Brandt and Lyubutina [1] observed in experiments aimed at revealing the de Haas - van Alphen effect in bismuth that clearly pronounced beats are observed in a number of cases.

The purpose of the present note is to present a possible explanation for the beats, on the basis of an analysis of the electron energy spectrum of doped bismuth placed in a magnetic field.

The main laws governing the observed effect are as follows: