mately

$$\frac{\hbar}{\tau} << \hbar\omega_H << \left(\frac{\hbar}{\tau}\frac{p_0^2}{2m}\right)^{1/2}. \quad (9)$$

which is attainable in a sufficiently pure material with not too large a nonlinearity.

Assuming that the dispersion law is as given by Kane [4], we get

$$\frac{p_0^2}{2m} = \frac{9}{64}\frac{\hbar^4 E_G^3}{m^2 P^4}, \quad (10)$$

where E_G is the width of the forbidden band and P is the matrix element of the spin-orbit interaction. To obtain an idea of the order of magnitude of the quantities, we assume numerical values for n-InSb (the quantities without dimensions correspond to atomic units):

$$E_G = 0{,}88\cdot 10^{-2} = 0{,}23\ \text{eV},\ P = 0{,}44,\ m = 1{,}3\cdot 10^{-2},$$
$$\tau = 10^{-11}\ \text{sec}$$

then

$$\frac{p_0^2}{2m} \simeq 2\cdot 10^{-2} \simeq 0{,}5\,\text{eV},\ \frac{\hbar}{\tau} \simeq 2\cdot 10^{-6} = 0{,}6\cdot 10^{-4}\ \text{eV}.$$

Assuming H = 500 Oe we get $\omega_H = 10^{12}$ sec^{-1} (λ = 2 mm) and $\hbar\omega_H = 0.6 \times 10^{-3}$ eV, so that (9) is satisfied. At these values we obtain $E_c \simeq 200$ V/cm.

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CAUSE OF BEATS IN THE OBSERVATION OF DE HAAS - VAN ALPHEN EFFECT IN METALS OF THE BISMUTH TYPE

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Brandt and Lyubutina [1] observed in experiments aimed at revealing the de Haas - van Alphen effect in bismuth that clearly pronounced beats are observed in a number of cases.

The purpose of the present note is to present a possible explanation for the beats, on the basis of an analysis of the electron energy spectrum of doped bismuth placed in a magnetic field.

The main laws governing the observed effect are as follows:

1. The beats are observed in the case low concentration of impurities of the donor type (for example, Te or Se in the Bi lattice).

2. The beat frequency depends on the type of impurity atoms, but is always proportional to the fundamental frequency of the oscillations.

We note also that the beats were observed in the cited reference in that part of the oscillatory field dependence of the magnetic field, which is due to the electronic part of the Fermi surface of bismuth.

The foregoing singularities can be understood by taking into account the deformation of the electron energy spectrum of the metal in the magnetic field under the influence of the impurities.

Passage of an electron near a positively-charged impurity atom can give rise not only to the local state observed by Yu. Bychkov [2] but also to a quasilocal state (see [3,4] concerning quasilocal levels). In the latter case, the quantum states of the electron correspond to particle motion near an impurity atom with rare transitions to one of the neighboring impurity atoms. Quantization of the electron orbits in the magnetic field leads to an aggregate of quasilocal levels alternating with the Landau levels.

The equation for the local and quasilocal levels, as is well known [4], is of the form

$$1 + U_0 F(E) = 0, \tag{1}$$

where F(E) is the real part of the Green's function of the electron in an ideal metal in the presence of a magnetic field, and U_0 is proportional to the scattering length. It is assumed in the derivation of this equation that the electron has quadratic dispersion and that the scattering potential of the impurity is δ-like. Of course, the impurity concentration, which is randomly distributed in the crystal lattice, is assumed small.

A detailed derivation and analysis of Eq. (1) will be the subject of a separate article by one of the authors. If we solve (1) graphically and recall that the density of the states unperturbed by the impurities decreases on approaching the Landau levels from the side of lower energies, then we arrive at the conclusion that quasilocal levels, alternating with the Landau levels, arise in the electron spectrum for any arbitrarily small positive U_0 (positively charged impurities). The number of the quasilocal levels between the neighboring Landau levels depends essentially on the form of the scattering potential, but the principal role will always be played by the quasilocal level closest to the Landau level (from the side of the lower energies), since it exhibits the smallest smearing. Therefore, in fact, the entire analysis is independent of the form of the scattering potential. We emphasize only that the closer the quasilocal level is to the Landau level, the narrower it is, and consequently, from this point of view, a small value of the parameter U_0 is "convenient." The quasilocal level becomes smeared out with increasing U_0.

The position of the quasilocal level is determined by the formula [5]

$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right) - \Delta,$$

$$\Delta = \frac{m^3\,\omega_c^2}{8\pi^2\hbar^4} U_0^2 \qquad (\Delta \ll \hbar\omega_c)$$

(m - effective mass of the electron, $\omega_c = eH/mc$ - cyclotron frequency).

The discrete aggregate of levels whose distances to the corresponding Landau levels depend nonlinearly on the magnetic field makes a positive oscillating contribution to the magnetic moment. The frequency Ω of these oscillations (the amplitude of which is proportional to the impurity concentration) is determined from the condition

$$\epsilon_F = \hbar\omega_c\left(n + \frac{1}{2}\right) - \Delta,$$

where ϵ_F is the Fermi energy. Hence

$$\Omega = \Omega_0\left(1 + \frac{\Delta}{\epsilon_F}\right).$$

Here $\Omega_0 = cS_{ext}/eh$ is the fundamental frequency [5].

Thus, the quasilocal levels lead to additional oscillations whose frequency does not differ much from Ω_0, since $\Delta \ll \epsilon_F$. Mixing of these oscillations with the fundamental ones produces beats with a frequency

$$\delta\Omega = \frac{\Delta}{\epsilon_F}\Omega_0$$

proportional to the oscillation frequency. The amplitude of the beats is proportional to the volume concentration of the impurity x and its order of magnitude is $x\rho^3$, where $\rho = \sqrt{c\hbar/eH}$ is the minimal magnetic length. The oscillation amplitude decreases with increasing impurity concentration. The entire picture becomes less distinct. At the same time, the quasilocal levels should become broader (this fact cannot be discerned in the linear theory) and, of course, the beats can no longer be resolved.

It is clear from (1) that Δ, meaning also the beat frequency, depends on U_0, i.e., on the type of impurity. In particular, the beat frequency increases with increasing U_0. Experimental investigations of the beats allow us to assess the character of the interaction between the electrons and the impurities.

It is clear from the foregoing that the beat picture described here is essentially connected with the sign of U_0. A system of quasilocal levels is produced only when $U_0 > 0$, corresponding to attraction of the electrons by the impurity atoms. In the experiments of Brandt and Lyubutina [1] the beats were observed when bismuth was doped with Te and Se which, according to present notions [1,6], behave like donors, i.e., become positively charged in the bismuth lattice.

Final confirmation of the hypothesis advanced here can be obtained by observing beats on the hole part of the field dependence of the momentum when acceptor impurities are introduced. The latter remark belongs to N. B. Brandt, to whom we are grateful for useful discussions.

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