not emit even optical lines.

4. However, we still did not take into account here the absorption, in the same zone, by meutral and ionized atoms and molecules (in particular, of the same kind as the emitter). In general the effect can take place in any medium with $\epsilon'' \neq 0$. It is therefore more convenient to express γ' more phenomenologically - in terms of the range of the photon absorption ℓ in the medium, $\exp(-\ell \operatorname{Im} k) \sim 1$, i.e., $\ell = (2|k|\xi)^{-1} \sim (|k|\epsilon'')^{-1}$. Equation (4) yields

$$\frac{\gamma'}{\gamma_0} = 2{,}7\,\frac{N^*\lambda_0^{*4}}{\ell}, \tag{8}$$

where ℓ is in centimeters. With such an approach N is no longer the number of electrons, but a parameter which determines, in accord with (2), the minimum distance, starting with which the collective quasistationary interaction is realized between the emitter and the absorbing particles (resonant atoms which lose in their excited state energy to friction and to impacts of the second kind, and also electrons producing the ohmic losses).

The conditions for the realization of the effect are worse in radioastronomic conditions, where N is relatively small. However, it may be encountered for rotational frequencies, and especially for vibrational frequencies, as well as for transitions between highly-excited levels.

It must be emphasized once more that owing to the strong dependence on the not-too-well-defined quantity R_0, the presented numerical estimate is quite rough. However, the effect itself is real. Thus, assuming, for the sake of reliability, a value of R_0 larger than (2) by 2 - 3 times, we obtain, to be sure, a smaller value of γ', but the correctness of the calculation becomes obvious. As noted by V. L. Ginzburg during the discussions, in the case of very small R_0 an important role may be assumed by spatial dispersion, which possibly automatically cuts off the integral, and the introduction of the parameter R_0 is unnecessary.

Indeed, if the Debye radius D turns out to be larger than R_0, then it can be assumed that one should choose as the lower limit in (1) not R_0 but D. In terms of our variables, $D/R_0 \sim 0.4T^{*1/2}N^{*-1/6}$. In such an approach, if $D/R_0 > 1$, we get in lieu of (7) $\gamma'/\gamma_0 \sim 90 \cdot L\lambda_0^{*6}N^{*7/2}/T^{*3}$. Of course, a more detailed analysis is required here.

I am grateful to G. A. Askar'yan, V. L. Ginzburg, D. A. Kirzhnits, and I. I. Sobel'man for useful discussions.

SINGULARITIES OF THE PHONON SPECTRUM OF CRYSTALS WITH EXTENDED DEFECTS

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Submitted 17 October 1967
ZhETF Pis'ma 7, No. 1, 32-36 (15 January 1968)

The purpose of the present note is to call attention to several very general peculiarities of the spectral functions of crystals with linear and two-dimensional defects (edge dislocations, stacking faults, flat boundaries, and others). If the defect has the form of a straight line or a plane (assuming that the defect concentration is low and they can be re-

garded as isolated), it causes violation of the translational symmetry of the crystal only in directions that are perpendicular to this line or plane. Therefore the component $f_{\parallel}$ of the wave vector parallel to the axis or to the plane of the defect is an integral of the motion, and the normal vibrations of the crystal take the form of waves $\varphi_{f_{\parallel},p}(x_{l\perp})\exp(if_{\parallel}x_{l\parallel})$ with frequencies $\omega(f_{\parallel}, p)$, where $x_{l\parallel}$ and $x_{l\perp}$ are respectively the components of the radius vector of the l-th particle in the direction parallel and perpendicular to the defect line or plane; p is the index numbering the normal vibrations for each $f_{\parallel}$ and assumes $3L_{\perp}$ values ("number of the branch"; $f_{\parallel}$ has $L_{\parallel}$ values); $L_{\parallel}$ and $L_{\perp}$ are the numbers of particles on the axis or on the plane of the defect and along the perpendicular cross section (plane or line), respectively, so that $L = L_{\parallel}L_{\perp}$ is the total number of particles in the system.

As shown by an analysis of simple models of extended defects (see [1,2]), among the indicated wave solutions there are as a rule some whose amplitude decreases exponentially with increasing distance from the defect: $\varphi_{f_{\parallel}p} \sim \exp(-x_{l\perp})/\rho$ ($\rho = \rho(f_{\parallel}, p)$ is the depth of penetration). However, it is possible to verify the existence of such solutions without resorting to simplified models. Far from the defect, the dynamic matrix corresponds to an ideal crystal whose normal vibrations are plane waves with frequencies $\omega_0(f, \sigma)$ (σ is the number of the branch). For each fixed $f_{\parallel}$ ($f = f_{\parallel} + f_{\perp}$) these frequencies have an upper and lower limit $\underline{\omega}_0(f_{\parallel})$ and $\bar{\omega}_0(f_{\parallel})$, respectively (we consider for simplicity a one-atom matrix without optical branches; the generalization to the case of an arbitrary matrix whose spectrum can have gaps can be effected directly), so that

$$\underline{\omega}_0(f_{\parallel}) \leqslant \omega_0(f_{\parallel}+f_{\perp}, \sigma) \leqslant \bar{\omega}_0(f_{\parallel}). \tag{1}$$

Let us constrain mentally all the atoms of the crystal, with the exception of those situated in the region of a defect, and let us excite in such a "bound" defect a natural oscillation with a certain $f_{\parallel}$ and with frequency $\omega_d(f_{\parallel}, p)$. For each $f_{\parallel}$ there can be $3r_{\perp}$ such waves ($p = 1, 2, \ldots, 3r_{\perp}$), where $r_{\perp}$ is the number of atoms in the perpendicular section of the defect (by a plane or by a line in the case of one- and two-dimensional defects, respectively). If it turns out that for a certain p we have

$$\omega_d(f_{\parallel}, p) < \underline{\omega}_0(f_{\parallel}) \quad \text{or} \quad \omega_d(f_{\parallel}, p) > \bar{\omega}_0(f_{\parallel}), \tag{2}$$

then, by virtue of (1), the indicated excitation cannot propagate through the crystals when the constraints are removed, and remains localized in a small region near the defect. This oscillation will occur at a somewhat altered frequency $\omega(f_{\parallel}, p)$, determined by solving exactly the dynamic problem, which satisfies as before one of the inequalities (2) ($\omega_d(f_{\parallel}, p) \to \omega(f_{\parallel}, p)$).

Thus, out of the $3L_{\perp}$ "branches" of the oscillations, a finite number ($\sim 3r_{\perp}$) of the "branches" may turn out to be localized, and the Van Hove theorem [4] can be used to analyze their contribution to the spectral functions $g^{ik}(\omega^2, x_{l\perp}^{(d)})$ and $g(\omega^2)$, which describe the dynamics of the individual atoms in the defect region and of the entire crystal as a whole, respectively (see [3]; $x_{l\perp} = x_{l\perp}^{(d)}$ correspond to the defect atoms). Using the results of this

theorem, which apply to one- and two-dimensional systems, we find directly that in the presence of local oscillations we have

$$g^{ik}(\omega^2, x_{\ell\perp}^{(d)}) = g_R^{ik}(\omega^2, x_{\ell\perp}^{(d)}) + g_J^{ik}(\omega^2, x_{\ell\perp}^{(d)}), \tag{3}$$

where $g_J^{ik}(\omega^2, x_{\ell\perp}^{(d)})$ has the following singularities in the vicinity of the critical points ω_{pI} of each of the local branches ($p = 1, 2, \ldots, \sim 3r_\perp$):

$$g_J^{ik}(\omega^2, x_{\ell\perp}^{(d)}) = C^{ik} |\omega^2 - \omega_{pI}^2|^{-1/2[1+(-1)^I \operatorname{sgn}(\omega^2 - \omega_{pI}^2)]}, \quad I = 0, 1 \tag{4}$$

in the case of linear defects and

$$g_J^{ik}(\omega^2, x_{\ell\perp}^{(d)}) = \begin{cases} \epsilon_I C^{ik} \operatorname{sgn}(\omega^2 - \omega_{pI}^2), \ \epsilon_0 = 1, \epsilon_2 = -1, \ I = 0, 2 & \text{a} \\ -C^{ik} \ln|\omega^2 - \omega_{pI}^2|, \quad I = 1 & \text{b} \end{cases} \tag{5}$$

in the case of two-dimensional defects (C^{ik} are certain constants). The function $g_R^{ik}(\omega^2, x_{\ell\perp}^{(d)})$ has singularities which are weaker by one order in $(\omega^2 - \omega_{pI}^2)$, or else singularities of the ordinary three-dimensional type [4] (generally speaking, at other points). The critical point $\omega_{pI} = \omega(f_{\|cr}, p)$ corresponds to the value $f_\| = f_{\|cr}$ at which the gradient $\nabla_{f\|}\omega^2(f_{\|cr}, p)$ vanishes for the p-local branch, and its index I is defined here as in [5]. The function $g(\omega^2)$ is represented in the same manner, except that its singular part (4) or (5) (C^{ik} is replaced by a scalar constant C) is proportional to the concentration of the defects.

The minimum number of critical points ω_{pI} of index I for each local branch is determined by the Morse theorem [4,5], with the exception of the singular points of index 0 of the acoustic local branches (i.e., those for which $\omega(f_\|, p) = cf_\|$ for small $f_\|$; their number does not exceed three), if such exist. The depth of penetration $\rho(f_\|, p)$ of the acoustic local oscillations in the case of small $f_\|$ is proportional to $f_\|^{-1}$ (compare with Rayleigh waves), so that these oscillations become strongly "collectivized" as $f_\| \to 0$. Therefore a singularity of the type (4) or (5a) in the vicinity of the minimum $\omega^2 = \omega^2(0, p) = 0$ ($I = 0$) does not appear and the spectral functions under consideration turn out to be proportional to $\sqrt{\omega^2}$, as is inherent in three-dimensional systems. We note in this connection that the presence of linear or planar defects in a crystal of larger dimensions, while the harmonic character of the interatomic forces remains valid in the entire crystal, cannot lead to a deviation of the lattice specific heat from the T^3 law when $T \ll \hbar\omega_{cr.min}$, where $\omega_{cr.min}$ is the critical point of type (4) or (5) which is closest to zero. The cases indicated, for example, in [5] are essentially caused by other factors (crystal of limited dimensions with secured boundaries, or thin plates). However, deviations can arise when $T \sim \hbar\omega_{cr.min}$ lies sufficiently low.

We note also that singularities of the type (4) and (5) are analogs of the δ-function singularities in the spectrum corresponding to the local oscillations of zero-dimensional

effects (say impurity atoms). However, unlike the latter (compare with quasilocal oscillations [6]), the singularities (4) and (5) become clearly manifest even if the case when the fall in the region of the continuous spectrum of the matrix $(0, \omega_{0max})$; ω_{pI} is only limited in that it must not belong to the region (1) (when $f_{\parallel} = f_{\parallel cr}$), which in general is much narrower than the band $(0, \omega_{0max})$.

Singularieties of type (4) or (5) should obviously be possessed by the functions of the state density of elementary excitations of arbitrary type, if localized states near extended defects are possible for these excitations.

The author is indebted to E. I. Rashba for a discussion of the work.

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FORMATION OF ULTRASHORT PULSES OF COHERENT LIGHT

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Submitted 24 October 1967
ZhETF Pis'ma 7, No. 1, 35-38 (15 January 1968)

1. Considerable progress was made recently towards the production of ultrasound pulses of coherent light by amplification in the nonlinear regime [1], and especially by linear absorption [2]. The laser of De Maria et al. [2] represents essentially a two-component laser medium, in which (unlike the two-component medium of [1]) the nonlinearly-absorbing component (saturable solution) effects compression of the pulse, and the amplifying component (neodymium glass) compensates for the unavoidable losses and allows generation to develop from the level of spontaneous noise. The purpose of the present letter is to examine the dynamics of the compression of the light pulses at maximum pulse-compression rate. Our results also describe the dynamics of light-pulse compression in a laser with self-phasing of modes by nonlinear absorption [2].

2. We shall consider the propagation of a light pulse in a two-component medium made of two-level absorbing and amplifying particles. The parameters of the medium and of the pulse satisfy the following conditions (the indices 1 and 2 pertain to the amplifying and absorbing particles, respectively):

$$T_1^{(1)} >> \tau_p >> T_1^{(2)}, T_2^{(1)}, T_2^{(2)}, \tag{1}$$

$$E_s^{(1)} >> E_s^{(2)}, E, \tag{2}$$

where τ_p and E are the duration and energy (in photons/cm^2) of the pulse, $E_s^{(1)} = \hbar\omega_0/2\sigma_i$